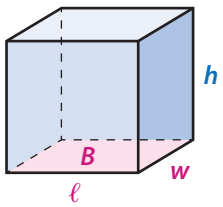
LESSON 11

Dear Family,

This week your student is learning how to find the volume of right rectangular prisms with fractional edge lengths.

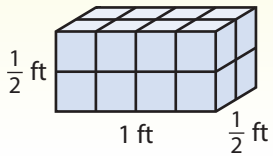
Previously, your student found volumes of prisms with whole-number edge lengths using the formulas below. These same formulas can be used to find volumes of prisms with fractional edge lengths.



$V = \ell wh$ and $V = Bh$

Your student will be learning to solve problems like the one below.

The right rectangular prism is filled with identical cubes. What is the volume of the prism?



➤ **ONE WAY** to find the volume is to use the volume of the cubes that fill the prism.

Four cubes fill the 1-ft length of the prism, so the edge length of each cube is $\frac{1}{4}$ ft.

Volume of each cube $= \ell wh$

$$= \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$$

The bottom layer of the prism has 8 cubes. There are 2 layers, so the total number of cubes is $8 \cdot 2$, or 16.

Multiply the **volume of one cube** by the **total number of cubes** inside the prism.

$$16 \cdot \frac{1}{64} = \frac{1}{4}$$

➤ **ANOTHER WAY** is to use the dimensions of the prism in a volume formula.

$$V = \ell wh$$
$$= 1 \cdot \frac{1}{2} \cdot \frac{1}{2}$$
$$= \frac{1}{4}$$

Using either method, the volume is $\frac{1}{4}$ ft^3.

Use the next page to start a conversation about volume.

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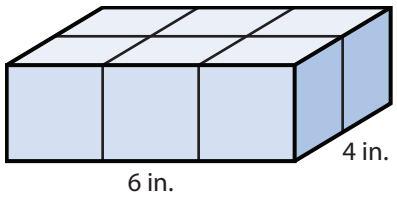
Activity Exploring Volume of Right Rectangular Prisms

➤ **Do this activity together to explore volume.**

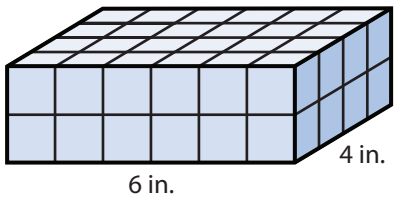
You can find the volume of a right rectangular prism using the number of identical cubes that you can pack inside of it.

What do you notice about these prisms?

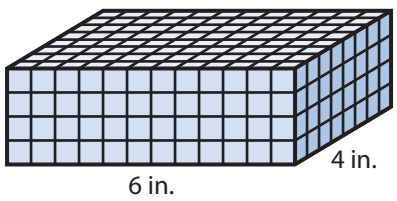
PRISM 1



PRISM 2



PRISM 3



? How does the size of the cube relate to the number of cubes needed to fill the prism?

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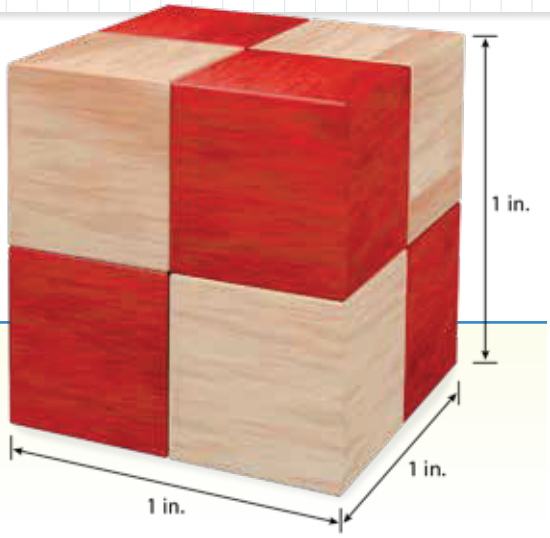
Explore Volume Problems with Fractions

Previously, you learned about the volume of right rectangular prisms with whole-number edge lengths. In this lesson, you will learn about the volume of right rectangular prisms with fractional edge lengths.

➤ **Use what you know to try to solve the problem below.**

Jiro has some small cubes. He puts them together to make a large cube, as shown. What is the volume of each small cube?



TRY IT

Math Toolkit grid paper, isometric dot paper, unit cubes

DISCUSS IT

Ask: How do you know the volume you found is reasonable?

Share: The volume makes sense because . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 8

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell wh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

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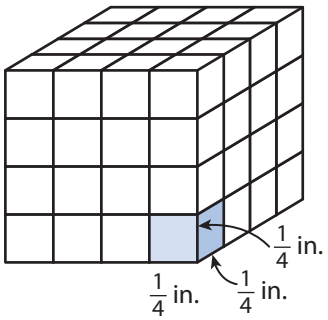
CONNECT IT

1 Look Back What is the volume of each small cube in Jiro's large cube? Explain.

2 Look Ahead You can use the volume of a unit cube to find the volume of a small cube with unit fraction edge lengths.



a. What is the edge length of the large cube shown here? Explain.

b. What is the volume of the large cube? How do you know?

c. How many small cubes make up the large cube? How do you know?

d. What does this tell you about the volume of one small cube? Why?

e. The formula $V = s^3$ gives the volume of a cube, where s is the edge length of the cube. Use the formula $V = s^3$ to find the volume of one small cube. Compare your answer here to your answer to problem 1d.

3 Reflect How is finding the volume of a cube with a fractional edge length similar to finding the volume of a cube with a whole-number edge length?

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 Name:

Prepare for Solving Volume Problems with Fractions

1 Think about what you know about three-dimensional figures and volume. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

In My Own Words

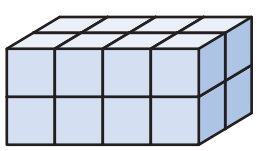
What I Know About It

volume

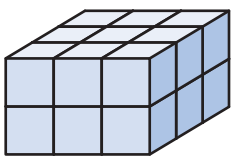
Examples

Examples

2 Prisms *A* and *B* are made of centimeter cubes. Which prism has a greater volume? Why?



Prism *A*



Prism *B*

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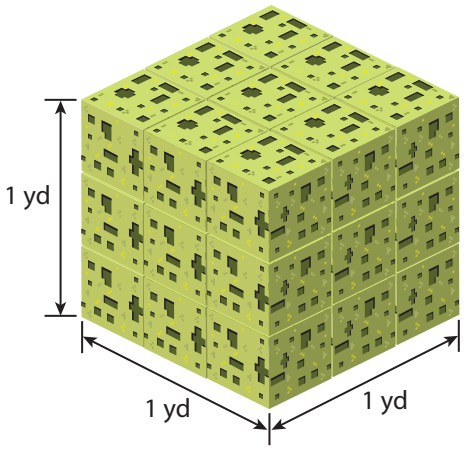
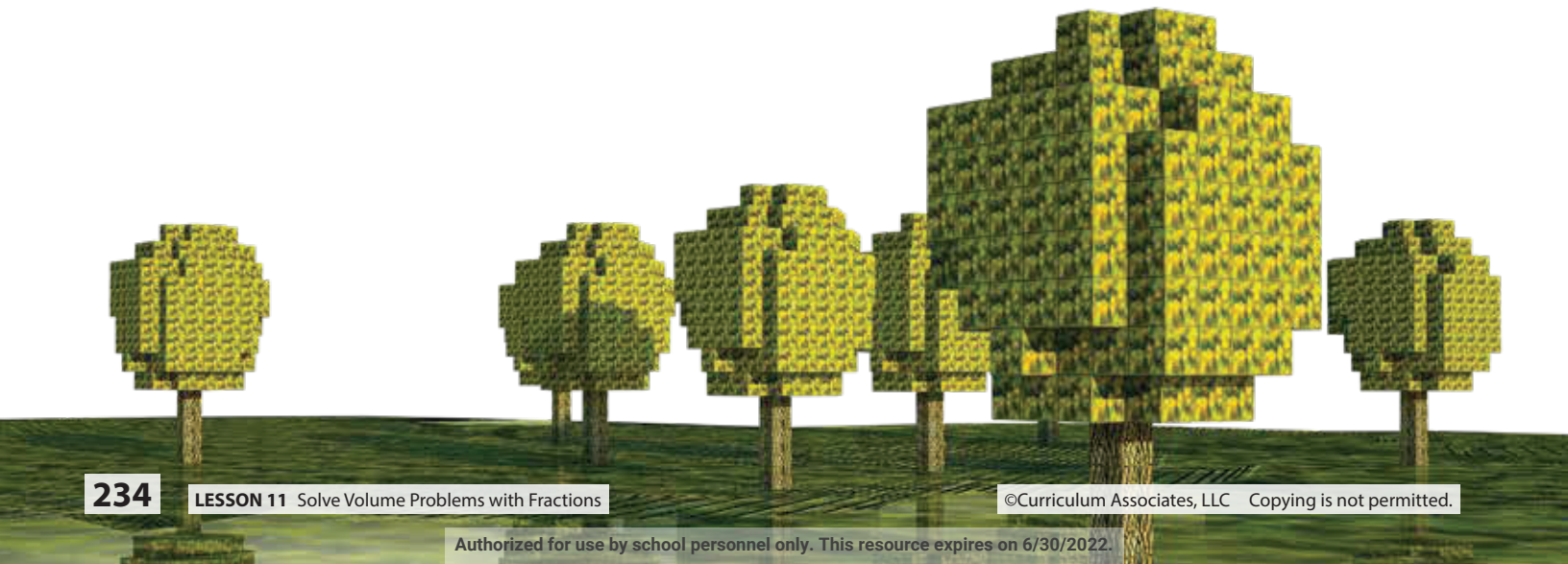
3 Isabella is playing a video game. Players use blocks to build objects. Isabella chooses blocks that are small cubes to build a large cube, as shown.



a. In the game, the edge length of the large cube is 1 yd. What is the volume of each small cube? Show your work.

SOLUTION _____

b. Check your answer to problem 3a. Show your work.



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Develop Finding the Volume of a Prism with Fractional Edge Lengths

Read and try to solve the problem below.

A reptile tank is a right rectangular prism. The tank is 1 ft long, $\frac{3}{4}$ ft wide, and $1\frac{1}{2}$ ft tall. What is the volume of the tank?

TRY IT

Math Toolkit grid paper, isometric dot paper, unit cubes

DISCUSS IT

Ask: How did you get started finding the volume?

Share: The first thing I did was . . .

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➤ **Explore different ways to find the volume of a right rectangular prism with fractional edge lengths.**

A reptile tank is a right rectangular prism. The tank is 1 ft long, $\frac{3}{4}$ ft wide, and $1\frac{1}{2}$ ft tall. What is the volume of the tank?

Model It

You can fill the prism with cubes with fractional edge lengths.

Find a fraction that divides 1, $\frac{3}{4}$, and $1\frac{1}{2}$ without a remainder. The edge length of each cube can be $\frac{1}{4}$ ft.

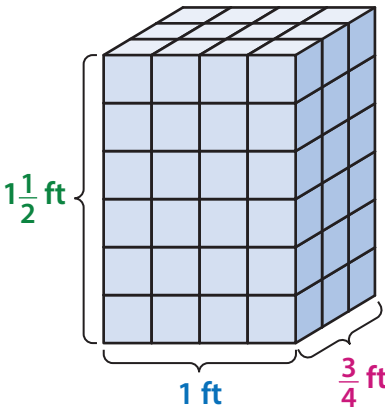
Cubes along length: $1 \div \frac{1}{4} = 4$

Cubes along width: $\frac{3}{4} \div \frac{1}{4} = 3$

Cubes along height: $1\frac{1}{2} \div \frac{1}{4} = 6$

Build a prism that is 4 cubes across, 3 cubes wide, and 6 cubes tall. The total number of cubes is $4 \times 3 \times 6 = 72$.

The volume of each cube is $\frac{1}{64}$ ft^3.

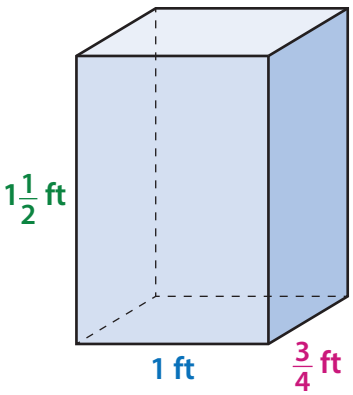


Model It

You can use a volume formula.

The volume V of a right rectangular prism is $V = \ell wh$.

$V = (1)\left(\frac{3}{4}\right)\left(1\frac{1}{2}\right)$



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CONNECT IT

Use the problem from the previous page to help you understand how to find the volume of a right rectangular prism with fractional edge lengths.

1. Look at the prism filled with cubes in the first **Model It**. Could you fill the prism with cubes that have edges that are $\frac{1}{2}$ ft long or $\frac{1}{3}$ ft long? Explain.

2. Why could you fill the prism with cubes that have edges that are $\frac{1}{8}$ ft long? How many cubes would fit along each edge of the prism?

3. Explain how to find the volume of a prism once you know the number of cubes that fill the prism. What is the volume of the reptile tank?

4. What is the volume of the tank using the formula $V = \ell wh$? How does it compare to the volume you get by filling the prism with cubes?

5. Describe two different ways to find the volume of a right rectangular prism with fractional edge lengths.

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the volume of a right rectangular prism with fractional edge lengths.

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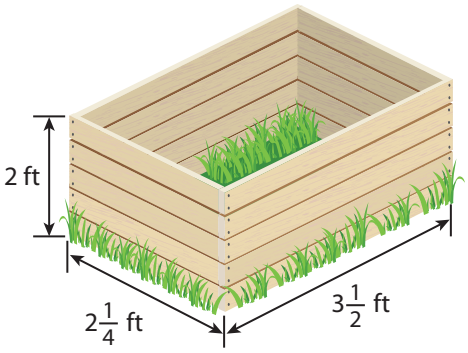
Apply It

Use what you learned to solve these problems.

7 Miyako is building a raised garden bed. The garden bed is a right rectangular prism with the dimensions shown. How many cubic feet of soil does Miyako need to fill the garden bed? Show your work.



SOLUTION

8 A right rectangular prism is completely filled with small cubes. Each small cube has edges that are $\frac{1}{4}$ in. long. The volume of the prism is 5 in.^{3}. How many cubes does it take to fill the prism?

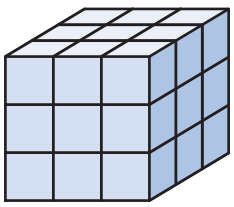
A 20

B 64

C 125

D 320

9 Each of the small cubes in this larger cube has edges that are $\frac{1}{6}$ yd long. What is the volume of the larger cube? Show your work.



SOLUTION

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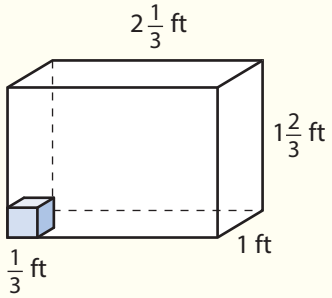
Practice Finding the Volume of a Prism with Fractional Edge Lengths

➤ **Study the Example showing how to find the number of cubes it takes to fill a prism. Then solve problems 1–4.**

Example

Geraldo has a collection of baseballs. He keeps each baseball in a cube-shaped box with edges that are $\frac{1}{3}$ ft long. Geraldo keeps the boxes in a rectangular bin, as shown. How many boxes can Geraldo keep in the bin?



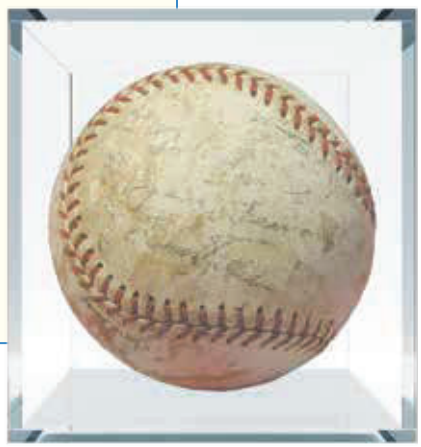
Divide to find the number of cube-shaped boxes that fit along the length, width, and height of the bin.

Length: $2\frac{1}{3} \div \frac{1}{3} = \frac{7}{3} \div \frac{1}{3} = 7$

Width: $1 \div \frac{1}{3} = 3$

Height: $1\frac{2}{3} \div \frac{1}{3} = \frac{5}{3} \div \frac{1}{3} = 5$

Since $7 \times 3 \times 5 = 105$, Geraldo can keep 105 boxes in the bin.



1 **a.** What is the volume of each cube-shaped box in the Example?

b. Use your answer to problem 1a to find the volume of the bin.

c. Show how you can use a formula to check the volume you found.

Vocabulary

cube
a rectangular prism in which each face of the prism is a square.

right rectangular prism
a right prism where the bases and other faces are rectangles.

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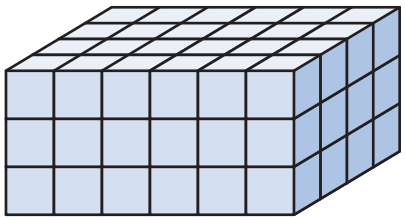
2 A fish pond at a park is in the shape of a right rectangular prism. The pond has a length of $3\frac{1}{2}$ yd, a width of $3\frac{1}{2}$ yd, and a height of $\frac{1}{4}$ yd. What is the volume of the pond? Show your work.

SOLUTION _____

3 A right rectangular prism has edge lengths of $3\frac{1}{2}$ ft, $2\frac{1}{4}$ ft, and $4\frac{1}{3}$ ft. Safara wants to completely fill the prism with cubes. The cubes must have edge lengths that are unit fractions. What is the greatest edge length of the cubes that Safara should use? Explain.

4 The right rectangular prism is filled with cubes. The edge length of each cube is $\frac{1}{2}$ in. What is the volume of the prism? Show your work.



SOLUTION _____

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Develop Solving Problems with Volume Formulas

➤ **Read and try to solve the problem below.**

Dalila bakes cornbread for family barbecues. The cornbread batter has a volume of 134 in.^{3}. She needs at least $1\frac{1}{4}$ in. of space between the top of the batter and the top of the pan. Will the batter fit in a rectangular pan that is 8 in. long, $12\frac{1}{2}$ in. wide, and 3 in. high?

TRY IT

Math Toolkit grid paper, isometric dot paper, unit cubes

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar to yours because . . . It is different because . . .

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➤ **Explore different ways to solve problems with volume formulas.**

Dalila bakes cornbread for family barbecues. The cornbread batter has a volume of 134 in.^{3}. She needs at least $1\frac{1}{4}$ in. of space between the top of the batter and the top of the pan. Will the batter fit in a rectangular pan that is 8 in. long, $12\frac{1}{2}$ in. wide, and 3 in. high?

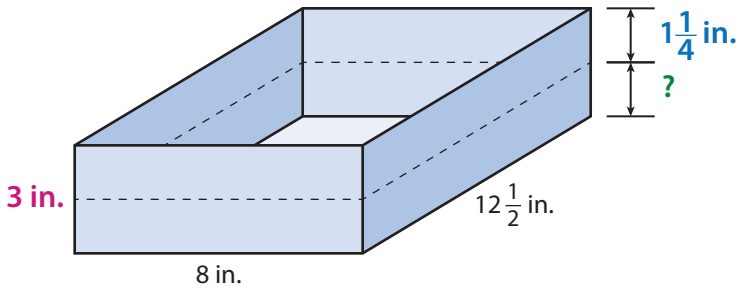
Picture It

You can draw a diagram to help you understand the problem.

Label the diagram with the length, width, and height of the pan.

The dashed line shows that Dalila needs at least $1\frac{1}{4}$ in. of space above the batter.

Because $3 - 1\frac{1}{4} = 1\frac{3}{4}$, the height of the batter can be no more than $1\frac{3}{4}$ in.



Model It

You can use a volume formula to help you solve the problem.

Find the volume of the space available for batter in the pan.

$$V = \ell wh$$
$$= (8)\left(12\frac{1}{2}\right)\left(1\frac{3}{4}\right)$$
$$= 175$$

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CONNECT IT

Use the problem from the previous page to help you understand how to solve problems with volume formulas.

1. Look at the diagram in **Picture It**. You can think of the dashed line as dividing the right rectangular prism of the pan into two smaller prisms. Describe what these two prisms represent in the context of the problem.

2. How does the diagram help you find the dimensions of the prism that represents the space available for the batter?

3. Will the batter fit in the pan with enough space left at the top? Explain.

4. Another prism represents the actual batter after it is poured into the pan. What is the volume of this prism? What dimensions of this prism do you know? Explain how you could find any missing dimensions.

5. Describe how you can approach solving a problem that involves the volume of one or more right rectangular prisms.

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

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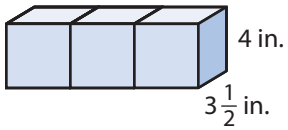
Apply It

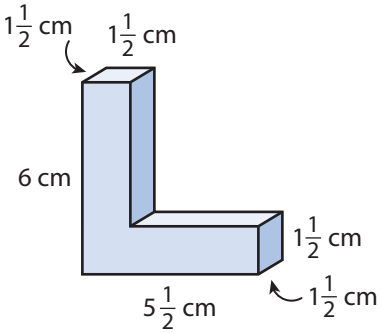
Use what you learned to solve these problems.

7 Roberto buys a package of three identical food containers, as shown. The total volume of the containers is 210 in.^{3}. How many of these containers could Roberto set side by side on a shelf that is 24 in. long? Show your work.



SOLUTION

8 What is the volume of the solid figure shown at the right?



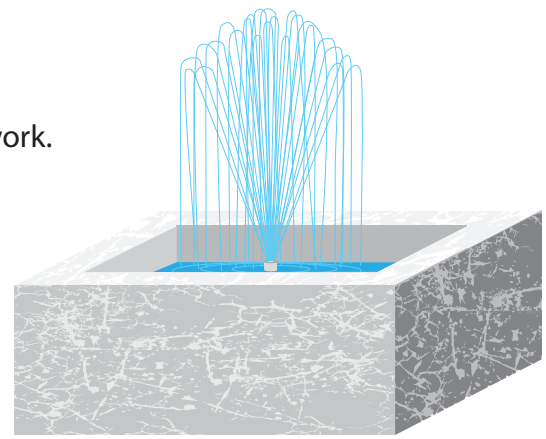
A $12\frac{3}{8}$ cm^3

B $13\frac{1}{2}$ cm^3

C $22\frac{1}{2}$ cm^3

D $25\frac{7}{8}$ cm^3

9 A fountain is a right rectangular prism that is $2\frac{1}{3}$ yd long, 3 yd wide, and $1\frac{1}{2}$ yd high. The fountain is $\frac{1}{3}$ full of water. What is the volume of the water in the fountain? Show your work.



SOLUTION

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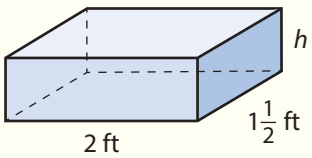
Practice Solving Problems with Volume Formulas

➤ **Study the Example showing how to solve a problem with volume formulas. Then solve problems 1–4.**

Example

A fish tank is a right rectangular prism that is 2 ft long and $1\frac{1}{2}$ ft wide. The tank can hold $3\frac{3}{4}$ ft^3 of water when it is full. Tameka fills the tank $\frac{2}{5}$ full of water. What is the height of the water in the tank?

Find the area of the base of the tank.



$$B = (2)\left(1\frac{1}{2}\right) = 3$$

Since $V = Bh$, you can divide the volume by B to find h.

$$h = 3\frac{3}{4} \div 3 = 1\frac{1}{4}$$

The height of the water when the tank is $\frac{2}{5}$ full is $\frac{2}{5}$ of h.

$$\frac{2}{5} \cdot 1\frac{1}{4} = \frac{2}{5} \cdot \frac{5}{4} = \frac{10}{20} = \frac{1}{2}$$

The height of the water in the tank is $\frac{1}{2}$ ft.

1 a. What is the volume of the water that Tameka puts in the fish tank in the Example? Explain how you can use the fact that the tank is $\frac{2}{5}$ full to find the volume of the water without finding the height of the fish tank.

b. How can you use your answer to problem 1a to find the height of the water? What is the height of the water?

Vocabulary

base (of a three-dimensional figure)
a face of a three-dimensional figure from which the height is measured.

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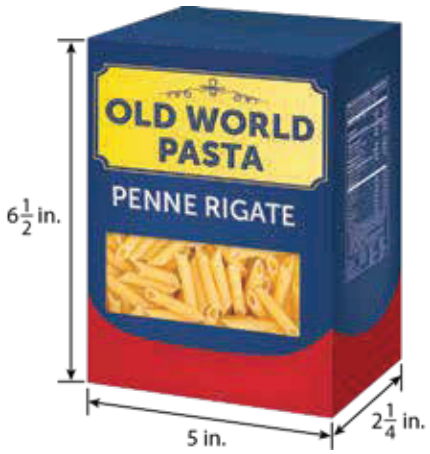
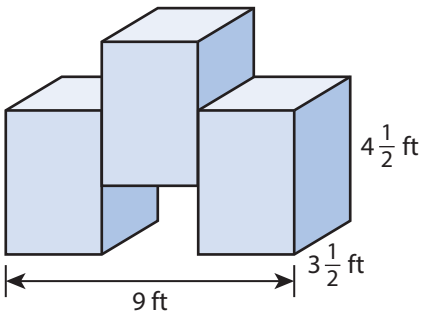
2 Kevin designs the pasta box shown. His box holds exactly the required amount of pasta. Kevin's boss says there must be at least $\frac{1}{2}$ in. of space between the top of the pasta and the top of the box. Kevin changes his design so that the height of the box is 9 in. and the area of the base is 9 in.^{2}. Will the pasta fit in the new box with enough space at the top? Explain.



3 A solid metal sculpture is made up of three identical right rectangular prisms. What is the volume of the metal? Show your work.



SOLUTION

4 A trash bin is a right rectangular prism with a width of $2\frac{1}{8}$ ft, a length of $1\frac{7}{8}$ ft, and a volume of $16\frac{1}{8}$ ft^3. Can a 3-ft board fit completely in the bin? Explain how to use estimation to solve the problem.

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Refine Solving Volume Problems with Fractions

➤ **Complete the Example below. Then solve problems 1–9.**

Example

A teacher stores number cubes in a rectangular box. The number cubes have edge lengths of 1.5 cm. The inside edge lengths of the box are 9 cm by 5.4 cm by 7.8 cm. What is the maximum number of cubes that can be stored in the box?

Look at how you could show your work using division of decimals.

Divide to find the maximum number of cubes that fit along the length, width, and height of the box.

Length: $9 \div 1.5 = 6$ ⟶ Exactly 6 cubes fit along the length.

Width: $5.4 \div 1.5 = 3.6$ ⟶ At most, 3 cubes fit along the width.

Height: $7.8 \div 1.5 = 5.2$ ⟶ At most, 5 cubes fit along the height.

Total number of cubes: $6 \times 3 \times 5$

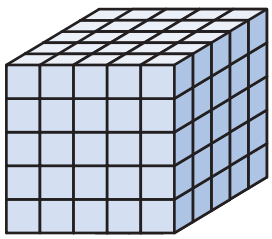
SOLUTION _____

CONSIDER THIS . . .
Can these cubes be used to fill the prism completely?

PAIR/SHARE
Could you solve the problem by finding the volume of the prism and dividing by the volume of a cube? Why or why not?

Apply It

1 Javier glues together small cubes to make a large cube, as shown. The large cube has edges that are $1\frac{1}{4}$ ft long. What is the volume of a small cube? Show your work.



SOLUTION _____

CONSIDER THIS . . .
How is the edge length of a small cube related to the edge length of the large cube?

PAIR/SHARE
How can you solve the problem a different way to check your answer?

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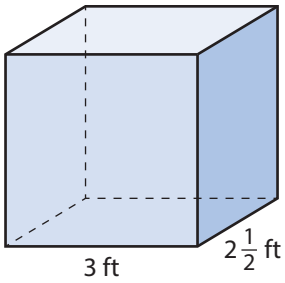
2 Ummi is setting up an aquarium in the shape shown here. She fills it partially with water. The volume of the water is 15 ft^3. Then she adds more water to the aquarium. Now the volume of the water is $22\frac{1}{2}$ ft^3. How much does the level of the water rise when Ummi adds more water? Show your work.

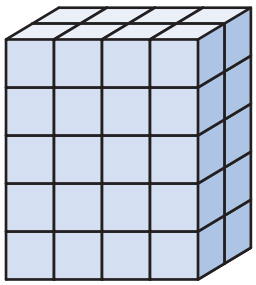


CONSIDER THIS . . .
If you know the volume of a rectangular prism and the area of the base, you can find the height.

PAIR/SHARE
Explain why the solution to the problem is not $22\frac{1}{2}$ ft − 15 ft = $7\frac{1}{2}$ ft.

SOLUTION

3 In the figure, each small cube has edges that are $\frac{1}{3}$ yd long. Which expression can you use to find the volume of the rectangular prism in cubic yards?



A $2 \times 4 \times 5$

B $2 \times 4 \times 5 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

C $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

D $2 \times 4 \times 5 \div \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right)$

Cody chose D as the correct answer. How might he have gotten that answer?

CONSIDER THIS . . .
How can you find the total number of small cubes in the rectangular prism?

PAIR/SHARE
What is another way you could write the expression that represents the volume of the rectangular prism?

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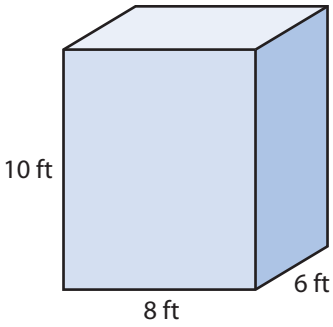
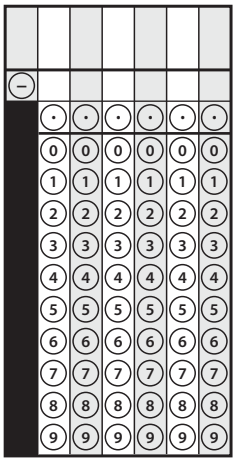
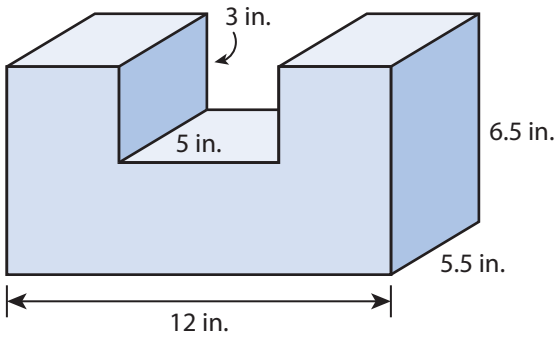
4 The figure shows the dimensions of a rectangular storage room. Darnell has boxes that are cubes with edges $1\frac{1}{2}$ ft long. What is the maximum number of boxes that Darnell can fit in the storage room? Explain.

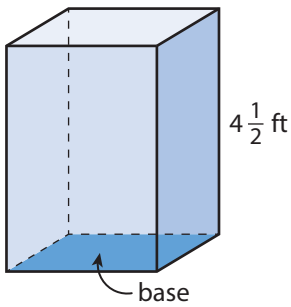


5 What is the volume of the solid figure in cubic inches?





6 It takes 144 cubes that have edge lengths $\frac{1}{2}$ ft to completely fill this cardboard box. What is the area of the base of the box?



A 4 ft^2

B 16 ft^2

C 18 ft^2

D 32 ft^2

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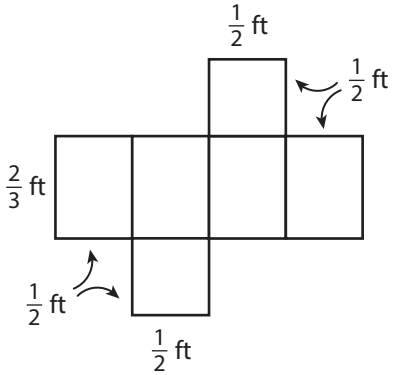
7 Dolores has a block of wax that is $2\frac{1}{2}$ in. long, 2 in. wide, and $4\frac{1}{2}$ in. high. She melts the wax and pours it into a candle mold. The mold is a right rectangular prism with a base area of $3\frac{3}{4}$ in.^{2}. What is the height of the wax in the mold? Show your work.

SOLUTION

8 What is the volume of the right rectangular prism shown by the net? Show your work.



SOLUTION

9 **Math Journal** Give the dimensions of a right rectangular prism that can be filled completely with cubes that have edge lengths of $\frac{1}{2}$ in. Explain how to use the cubes to find the volume of the prism.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *diagram*. Write at least one synonym for *diagram*.
- ☐ **SELF CHECK** Go back to the Unit 2 Opener and see what you can check off.

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