Understand Multiplication as Scaling

LESSON 21

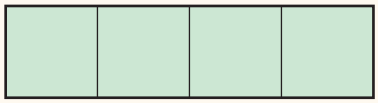
Dear Family,

This week your child is exploring multiplication as scaling.

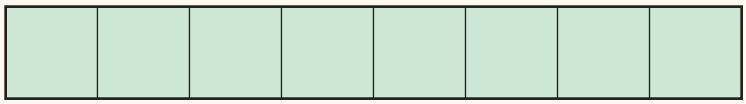
Scaling is resizing a quantity through multiplication. You can think of scaling as stretching or shrinking.

You can *stretch*, or increase, a quantity by multiplying the quantity by a factor greater than 1. You can *shrink,* or decrease, a quantity by multiplying the quantity by a factor less than 1.

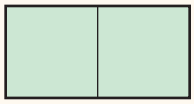
Look at the length of the bar below. It has a length of 4 units.



If you multiply the length by 2, you double the length of the bar.



If you multiply the original length by $\frac{1}{2}$, you shrink the bar to half its original length.



Your child is learning to generalize about multiplication and scaling. Multiplying by a number . . .

- greater than 1 increases the quantity.
- less than 1 decreases the quantity.
- equal to 1, such as $\frac{4}{4}$, means that the quantity stays the same.

Invite your child to share what he or she knows about multiplication as scaling by doing the following activity together.

ACTIVITY MULTIPLICATION AS SCALING

Do this activity with your child to understand multiplication as scaling.

Use the examples below to talk with your child about multiplication as scaling.

- This is the actual size of a pencil that is 4 centimeters long.

- Ask your child the following questions.

1. *What if the pencil were twice as long? How long would it be? How do you know?*

2. *What if the pencil were half as long as the original pencil? How long would it be? How do you know?*

3. *What if the pencil were 3 times as long? Would it be shorter or longer than the original pencil? How do you know?*

4. *What if the pencil were $\frac{3}{4}$ as long? Would it be shorter or longer than the original pencil? How do you know?*

5. *What if the pencil were $\frac{4}{4}$ as long? How would the length of the pencil compare to the length of the original pencil?*

6. *What would it mean to multiply the length of the pencil by $\frac{7}{4}$? How would the length of the pencil change?*

Answers:

1. It would be two times the length of the original pencil, or 8 centimeters. It would be longer than the original pencil because we multiplied by a number greater than 1.

2. It would be half the length of the original pencil, or 2 centimeters. It would be shorter than the original pencil because we multiplied by a number less than 1.

3. It would be longer because we are multiplying by a number greater than 1.

4. It would be shorter because we are multiplying by a number less than 1.

5. It would be the same length as the original pencil because we are multiplying by a number equal to 1.

6. It would be longer because we are multiplying by a number greater than 1.

Explore Multiplication as Scaling

What does scaling mean?

Learning Target

- Interpret multiplication as scaling (resizing), by:
  - Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

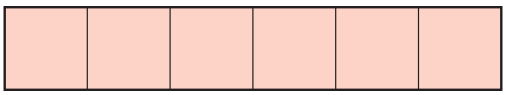
SMP 1, 2, 3, 4, 5, 6, 7, 8

MODEL IT

Complete the problems below.

1 Changing the size of a quantity by multiplication is called **scaling**. *Stretching* and *shrinking* are two different ways to scale a quantity.

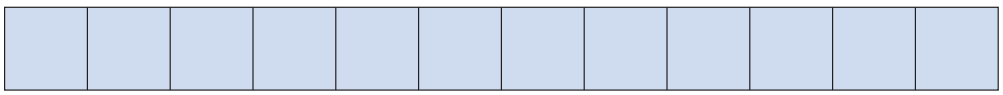
This bar has a length of 6 units.



Use the bars at the bottom of the page to complete parts a and b.

a. Circle the bar that shows the length of 6 units being doubled, or stretched. Underline the bar that shows the length of 6 units being halved, or shrunk.

b. Write a multiplication equation for each bar. Circle the factor that describes how the length of 6 units has been stretched or shrunk.



Equation .....

Equation .....

DISCUSS IT

- Did you and your partner agree on which bar shows shrinking and which shows stretching?
- I think you can double the value of a quantity by multiplying by . . .
  I think you can halve the value of a quantity by multiplying by . . .

MODEL IT

Complete the problems below.

2 Use the table to make generalizations about scaling the whole number 6 through multiplication by various factors.

Factor	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{4}{4}$	$\frac{3}{2}$	$2\frac{1}{2}$	3
Factor	6	6	6	6	6	6	6	6
Product	$\frac{6}{10}$	2	3	6	6	9	15	18

a. Underline the products less than 6. What do the factors for these products have in common?

b. Circle the products greater than 6. What do the factors for these products have in common?

c. Put a box around the products equal to 6. What do the factors for these products have in common?

DISCUSS IT

- How did you and your partner describe what the products less than, greater than, and equal to 6 have in common?
- I think when you multiply by a fraction less than 1, the product will be . . .

  I think when you multiply by a fraction greater than 1, the product will be . . .

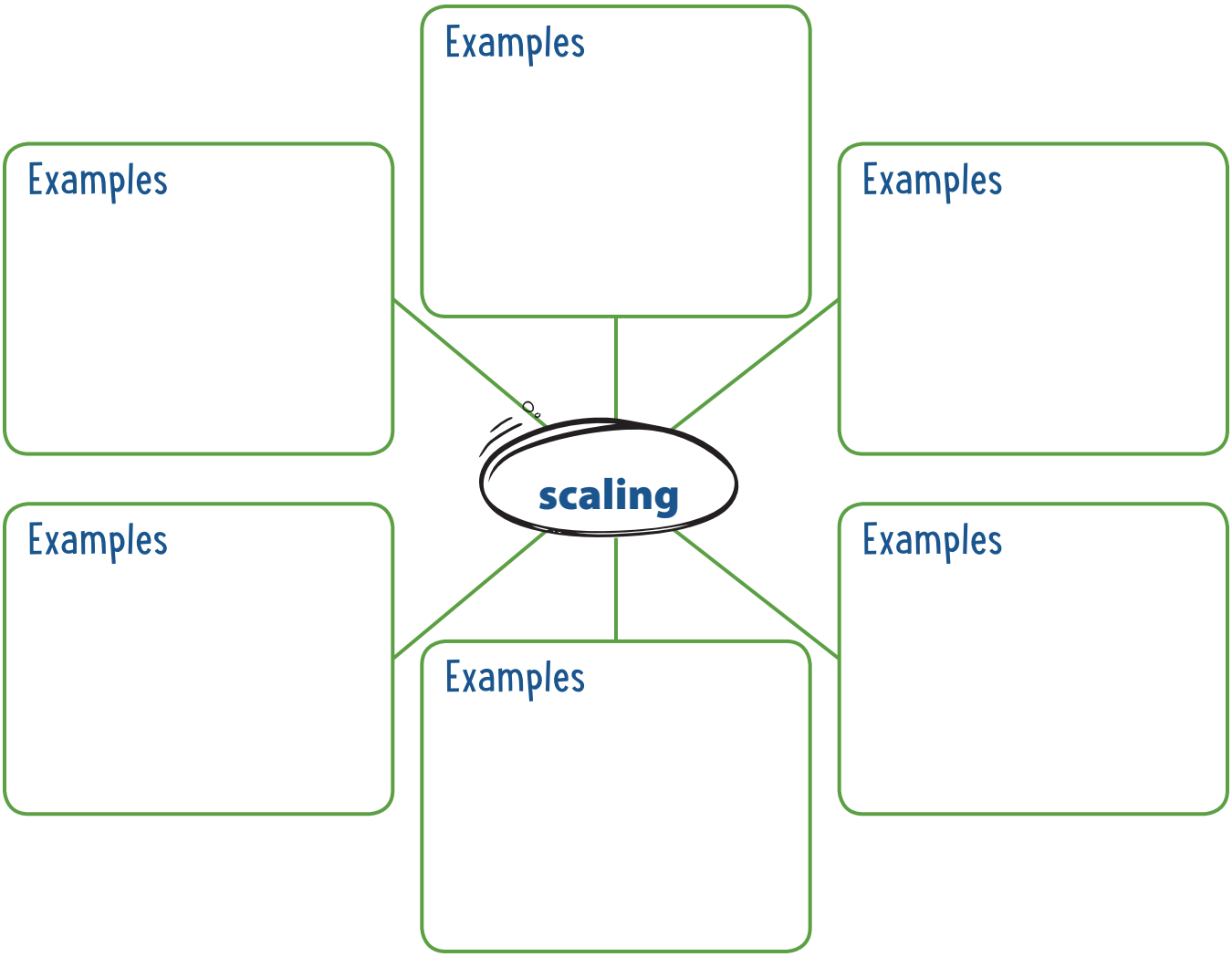
  I think when you multiply by a fraction equal to 1, the product will be . . .

3 REFLECT

Describe the products you get if you multiply 8 by factors less than 1. Describe the products you get if you multiply 8 by factors greater than 1. Give some examples that justify your answers.

Prepare for Multiplication as Scaling

1 Think about what you know about multiplying by numbers greater than or less than 1. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



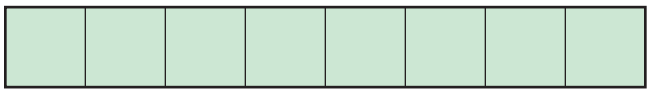
2 Complete the sentences to describe two kinds of scaling.

Shrinking: When I multiply the whole number 6 by a fraction less than 1, the product will be _____.

Stretching: When I multiply the whole number 6 by a fraction greater than 1, the product will be _____.

Solve.

This bar has a length of 8 units.

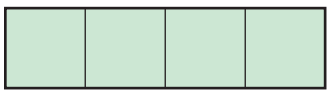


3 This bar shows the length of the 8 units scaled.

a. Use words to describe how the length of 8 units is scaled.

b. Write a multiplication equation to show how the length is scaled.

4 This bar shows the length of 8 units scaled in a different way.



a. Use words to describe how the length of 8 units is scaled.

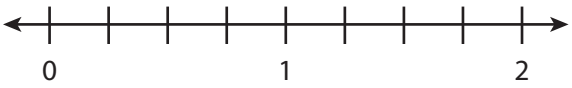
b. Write a multiplication equation to show how the length is scaled.

Develop Understanding of Multiplication as Scaling

MODEL IT: NUMBER LINES

Try these three problems.

1 The first number line shows $\frac{3}{4}$. Show $\frac{1}{3} \times \frac{3}{4}$ on the second number line.

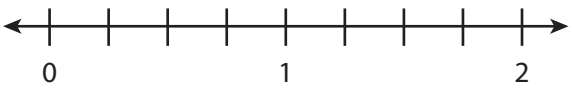


Is $\frac{1}{3} \times \frac{3}{4}$ *less than, greater than,* or *equal to* $\frac{3}{4}$? .....

2 Use the number line to show $\frac{5}{3} \times \frac{3}{4}$.

Is $\frac{5}{3} \times \frac{3}{4}$ *less than, greater than,* or *equal to* $\frac{3}{4}$? .....

3 Use the number line to show $\frac{3}{3} \times \frac{3}{4}$.



Is $\frac{3}{3} \times \frac{3}{4}$ *less than, greater than,* or *equal to* $\frac{3}{4}$? .....

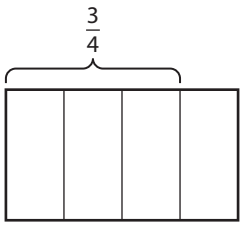
DISCUSS IT

- Did you and your partner model $\frac{5}{3} \times \frac{3}{4}$ in the same way?
- I think one of these products is less than $\frac{3}{4}$, one is equal to $\frac{3}{4}$, and one is more than $\frac{3}{4}$ because . . .

MODEL IT: AREA MODELS

Use an area model to show scaling.

4 Shade the area model to show $\frac{1}{3} \times \frac{3}{4}$.



Is $\frac{1}{3} \times \frac{3}{4}$ *less than, greater than,* or *equal to* $\frac{3}{4}$? .....

DISCUSS IT

- Did you and your partner model $\frac{1}{3} \times \frac{3}{4}$ in the same way?
- I think $\frac{7}{5} \times \frac{3}{4}$ is greater than $\frac{3}{4}$ because . . .

CONNECT IT

Complete the problems below.

5 In problem 1, you modeled $\frac{1}{3} \times \frac{3}{4}$ on a number line, and in problem 4 you used an area model to show the same product. Do both models show the same comparison between $\frac{1}{3} \times \frac{3}{4}$ and $\frac{3}{4}$? Explain.

6 Choose any model you like to show how the product $\frac{3}{2} \times \frac{4}{3}$ compares to $\frac{4}{3}$. Then complete the comparison.

$\frac{3}{2} \times \frac{4}{3}$ is ..... $\frac{4}{3}$.

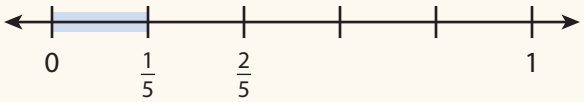
Practice Multiplication as Scaling

Study the Example showing how to use a number line to multiply by fractions less than 1 and greater than 1. Then solve problems 1–6.

EXAMPLE

Multiply $\frac{2}{5}$ by a fraction less than 1. $\frac{1}{2} \times \frac{2}{5} = ?$

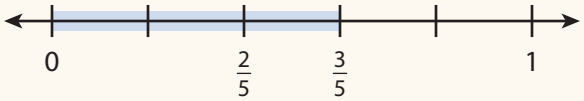
Break $\frac{2}{5}$ into 2 equal parts. Each part is $\frac{1}{5}$. Shade 1 part.



$\frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

Multiply $\frac{2}{5}$ by a fraction greater than 1. $\frac{3}{2} \times \frac{2}{5} = ?$

Break $\frac{2}{5}$ into 2 equal parts. Each part is $\frac{1}{5}$. Shade 3 parts.



$\frac{3}{2} \times \frac{2}{5} = \frac{3}{5}$

1 When you multiply a whole number by a fraction less than 1, the product is less than the whole number. Does the Example showing $\frac{1}{2} \times \frac{2}{5}$ support a similar rule when multiplying a fraction by a fraction less than 1? Explain.

2 Use the Example showing $\frac{3}{2} \times \frac{2}{5}$. Which statements correctly describe how the product compares to one of its factors?

Ⓐ $\frac{3}{2} \times \frac{2}{5}$ is one and one-half times $\frac{2}{5}$.

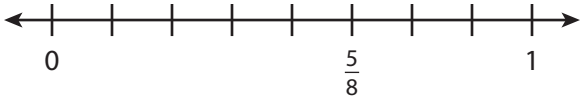
Ⓑ $\frac{3}{2} \times \frac{2}{5}$ is equal to $\frac{3}{2}$.

Ⓒ $\frac{3}{2} \times \frac{2}{5}$ is less than $\frac{2}{5}$.

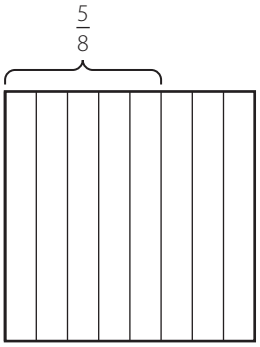
Ⓓ $\frac{3}{2} \times \frac{2}{5}$ is greater than $\frac{3}{2}$.

Ⓔ $\frac{3}{2} \times \frac{2}{5}$ is greater than $\frac{2}{5}$.

3 Shade the number line to show $\frac{3}{5} \times \frac{5}{8}$.



4 Complete the area model to show $\frac{3}{5} \times \frac{5}{8}$.



5 Look at your models in problems 3 and 4. Is $\frac{3}{5} \times \frac{5}{8}$ greater than or less than $\frac{5}{8}$? Is it easier to compare the product to $\frac{5}{8}$ using the number-line model or the area model? Explain.

6 Choose the correct word to fill in each blank below.

a. When you multiply a given fraction by a fraction equal to 1, the product will ____ be equal to the original fraction.

Ⓐ never Ⓑ sometimes Ⓒ always

b. When you multiply a given fraction by a factor less than 1, the product will ____ be greater than the given fraction.

Ⓐ never Ⓑ sometimes Ⓒ always

c. When you multiply a given fraction by a factor greater than 1, the product will ____ be less than 1.

Ⓐ never Ⓑ sometimes Ⓒ always

Refine Ideas About Multiplication as Scaling

APPLY IT

Complete these problems on your own.

1 ANALYZE

Use reasoning to order the following expressions from least to greatest. Do not calculate any of the products. Explain your reasoning.

$\frac{7}{9} \times 348{,}980$ $\frac{12}{11} \times 348{,}980$ $\frac{50}{50} \times 348{,}980$

2 INTERPRET

Two towns are comparing populations. Town A has 285,310 residents, and Town B has $\frac{9}{10} \times 285{,}310$ residents.

To compare the populations of the two towns, Jonah said the population of Town A is nine tenths of the population of Town B. Is he correct? Explain your answer.

3 EXPLAIN

Gillian said that the product of a given number and a fraction is always less than the given number. Is Gillian correct? Explain and give an example.

PAIR/SHARE

Discuss your solutions for these three problems with a partner.

Use what you have learned to complete problem 4.

4 You can compare the size of a product to the size of the factors in a multiplication equation if you know whether the factors are greater than, less than, or equal to 1.

Part A Write a multiplication equation (different than any in this lesson) in which the product is greater than both of its factors. At least one factor should be a fraction. Draw a model to support your answer.

Solution

Part B Write a multiplication equation (different from any in this lesson) in which both factors are fractions and the product is less than both of its factors. Draw a model to support your answer.

Solution

5 MATH JOURNAL

How does $\frac{4}{4} \times \frac{8}{5}$ compare to $\frac{8}{5}$? Explain your reasoning.