**Analyze Two-Variable Relationships**

## Dear Family,

This week your student is learning how to analyze the relationship between two quantities that change. The value of one quantity, called the **dependent variable**, depends upon the value of the other quantity, called the **independent variable**.

For example, the equation $d = 1.25t$ represents the distance, $d$, in miles traveled by a train in $t$ minutes. The **distance** traveled depends on how much **time** has passed.

$$d = 1.25t$$

**dependent variable**   **independent variable**
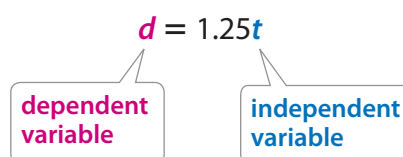
Your student will be learning to solve problems like the one below.

The equation $y = 5x$ represents the total cost, $y$, in dollars to rent a bicycle for $x$ hours. How much does the total cost increase for each additional hour a person rents the bicycle?

➤ **ONE WAY** to analyze the relationship between two variables is to use a table.

| Time (hours), x | Total Cost ($), y |
|---|---|
| 0 | 0 |
| 1 | 5 |
| 2 | 10 |
| 3 | 15 |
| 4 | 20 |

Each time the number of hours increases by 1, the total cost increases by $5.

➤ **ANOTHER WAY** is to graph the ordered pairs $(x, y)$.
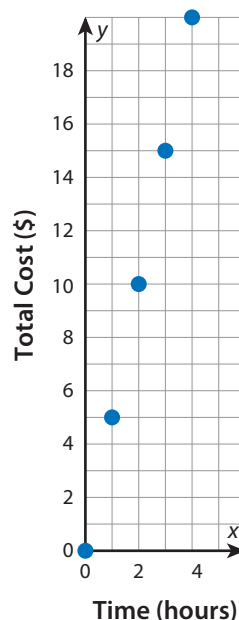
Using either method, the total cost increases by $5 for each additional hour.

▶ Use the next page to start a conversation about two-variable relationships.

# Activity Thinking About Two-Variable Relationships Around You

➤ **Do this activity together to investigate two-variable relationships in the real world.**

Did you know that the amount of time you spend sleeping can depend on how long it takes you to get to school?

For each minute you spend commuting to school, you can expect to lose an average of 1.3 minutes of sleep. This relationship can be represented with the equation $y = 1.3x$, where $x$ represents commuting time in minutes and $y$ represents minutes of lost sleep.

**1 minute of commuting results in 1.3 minutes of lost sleep**

**?** Where else do you see two-variable relationships in the world around you?

# Explore Two-Variable Relationships

Previously, you learned about equations with one variable. In this lesson, you will learn about equations with two variables.

➤ **Use what you know to try to solve the problem below.**

Hiroko makes amigurumi animals for her cousins. The instructions for making an amigurumi fox say to crochet the stitches in a row and then join the ends to make a "round." How can Hiroko use the round number to determine the number of stitches in that round?

**Round 1:** 6 stitches
**Rounds 2 to 7:** Increase the number of stitches in the previous round by 6.

**TRY IT**

🖊 **Math Toolkit** connecting cubes, counters, grid paper

**DISCUSS IT**

*Ask:* How could Hiroko use the relationship you found to determine the number of stitches in Round 7?

*Share:* To determine the number of stitches in Round 7, Hiroko could . . .

◎ **Learning Target**    SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

## CONNECT IT

**1** **Look Back** How can Hiroko use the number of the round to determine the number of stitches in a round of her fox pattern? Explain how you know.

**2** **Look Ahead** In Hiroko's pattern, the number of stitches in a round depends on the round number. An equation in two variables is one way to represent the relationship between two quantities. The value of the **dependent variable** depends on the value of the **independent variable**.

**a.** A store sells kits for making amigurumi animals. Each kit costs $15. The equation $c = 15k$ can be used to determine the total cost, $c$, of $k$ kits. In this situation, is $c$ or $k$ the dependent variable? Explain.

**b.** The equation $k = \frac{c}{15}$ can be used to find the number of kits, $k$, you can buy with $c$ dollars. In this situation, is $c$ or $k$ the dependent variable? Explain.

**c.** How are the equations in problems 2a and 2b alike? How are they different?

**3** **Reflect** Hiroko and her cousins have $45 to spend on kits. Which equation do you think they would use to find the number of kits they can buy? Why?

# Prepare for Analyzing Two-Variable Relationships

**1** Think about what you know about variables, expressions, and equations. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

**Examples**

**Examples**

**Examples**

**Examples**

**Examples**

**Examples**
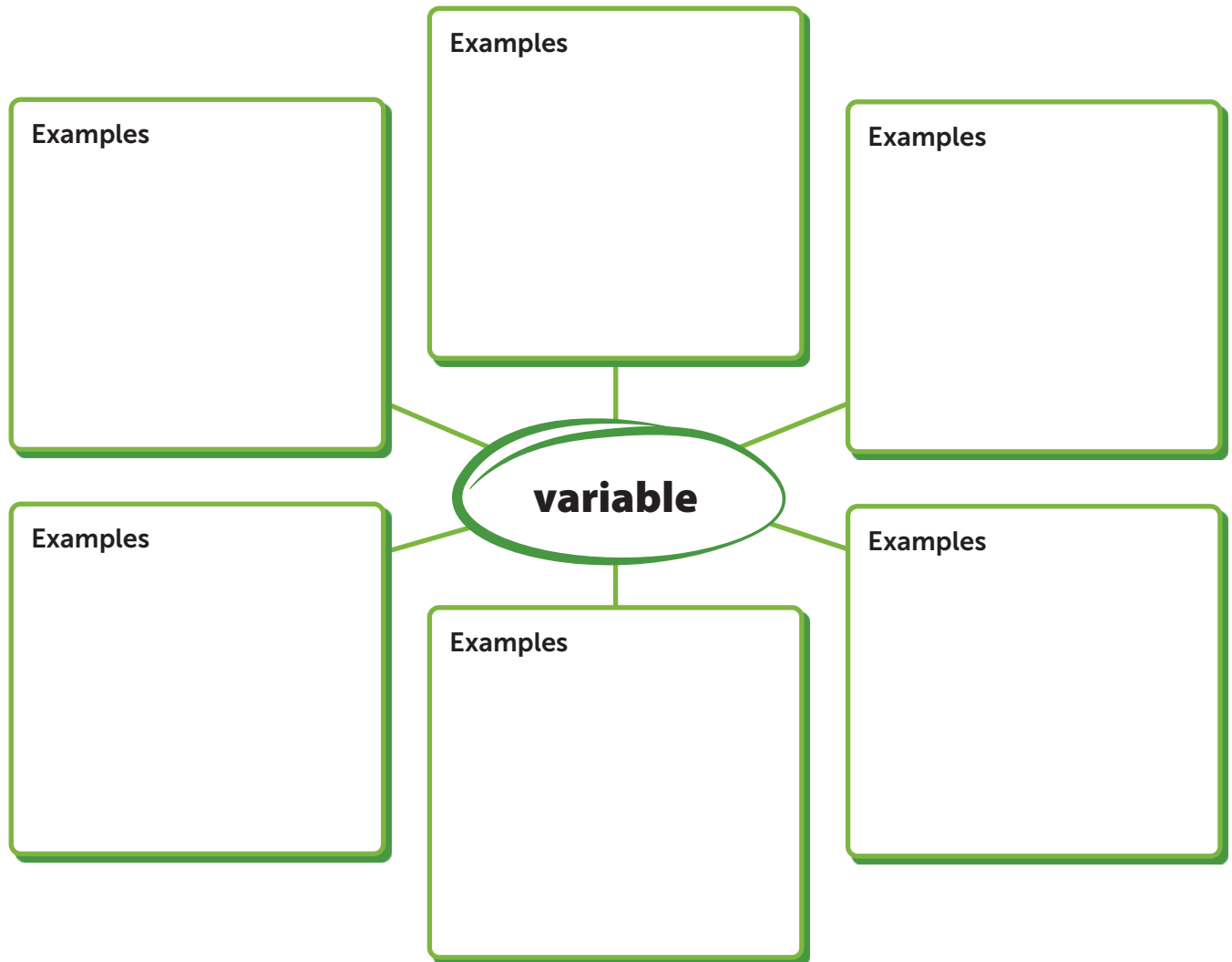
**variable**

**2** Why can the value of the variable $a$ in the expression $a + 2$ be any number?
Why does the variable $b$ in the equation $b + 2 = 5$ have only one value?

**3** There are 5 hidden treasures in Level 1 of a smartphone game. Each of the other levels has 1 more hidden treasure than the previous level.

**a.** How can you use the level number in the game to determine the number of hidden treasures in that level? Show your work.

SOLUTION _____

**b.** Check your answer to problem 3a. Show your work.

# Develop Writing an Equation in Two Variables to Represent a Relationship

➤ **Read and try to solve the problem below.**

An animal reserve is home to 8 meerkats. It costs the reserve $1.50 per day to feed each meerkat. Write an equation with two variables that can be used to determine the total cost of feeding the reserve's meerkats for any number of days.

**MEERKAT HABITAT**

Population
**8 meerkats**

Cost per meerkat
**$1.50 per day**

**TRY IT**

**Math Toolkit** counters, grid paper, sticky notes

**DISCUSS IT**

***Ask:*** What does each variable in your equation represent?

***Share:*** In my equation, the variable . . . represents . . .

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➤ **Explore different ways to write an equation in two variables to represent a relationship.**

An animal reserve is home to 8 meerkats. It costs the reserve $1.50 per day to feed each meerkat. Write an equation with two variables that can be used to determine the total cost of feeding the reserve's meerkats for any number of days.

## Model It

You can make a table to look for a relationship between two quantities that vary.

The two quantities are the **number of days** and the **total cost to feed 8 meerkats**.

$d$ = number of days

$t$ = total cost ($) to feed 8 meerkats

In each row, the total cost is **12** times the number of days.

| Number of Days, $d$ | $(d \times 1.5) \times 8$ | Total Cost ($), $t$ |
|---|---|---|
| 1 | $(1 \times 1.5) \times 8$ | 12 |
| 2 | $(2 \times 1.5) \times 8$ | 24 |
| 3 | $(3 \times 1.5) \times 8$ | 36 |
| 4 | $(4 \times 1.5) \times 8$ | 48 |

$\times$ **12**

## Model It

You can use words to help you write an equation in two variables.

It costs $1.50 per day to feed 1 meerkat.

$8 \times 1.50 =$ **12**

So, it costs **$12 per day** to feed 8 meerkats.

Describe how to use the number of days, $d$, to find the total cost, $t$.

The **total cost** equals the **cost per day** times the **number of days**.

## CONNECT IT

➤ **Use the problem from the previous page to help you understand how to write an equation in two variables to represent a relationship.**

**1** Look at the variables $d$ and $t$ in the first **Model It**. Which variable is the independent variable, and which is the dependent variable? How do you know?

**2** The table shows the expression $(d \times 1.50) \times 8$. How does this expression represent finding the total cost to feed 8 meerkats?

**3** How is the factor 12 shown below the table related to $(d \times 1.5) \times 8$? What does 12 represent in this situation?

**4** Look at the second **Model It**. What equation can you use to find the total cost of feeding the meerkats for any number of days? Use your equation to find the total cost of feeding the meerkats for 1 year, or 365 days.

**5** Why is an equation a good model for representing the relationship between an independent variable and a dependent variable?

**6** **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to write an equation in two variables to represent a relationship.

## Apply It

➤ **Use what you learned to solve these problems.**

**7** A company makes frames for photographs. The height of each frame is 2 inches greater than the height of the photo it is designed to hold.

*f* in.          *p* in.

**a.** Write an equation that shows how to find the height, *f*, in inches of a frame designed to hold a photo with a height of *p* inches. Show your work.
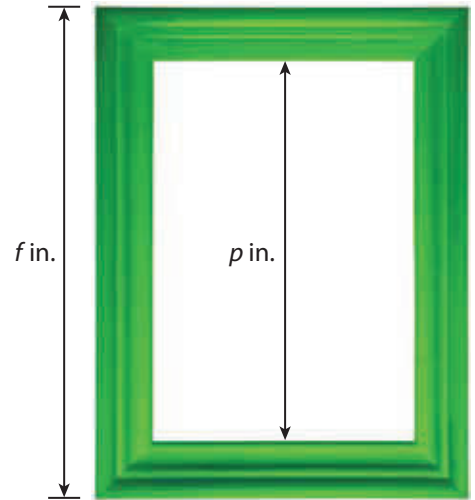
**SOLUTION** _____

**b.** Which variable is the dependent variable in your equation? How do you know?

**8** Snow is falling at a constant rate. The table shows how much snow has fallen since the snow began. Alexis writes the equation $y = x + \frac{1}{4}$ to represent the relationship between the time in hours and the inches of snow. Does her equation correctly model the relationship? Explain.

| Time (h), *x* | Snow (in.), *y* |
|:---:|:---:|
| 1 | $1\frac{1}{4}$ |
| 2 | $2\frac{1}{2}$ |
| 3 | $3\frac{3}{4}$ |
| 4 | 5 |

**9** A company makes several sizes of phones. For each size, the ratio of the height of the screen to its width is 18 : 9. Write an equation that shows how to find the height in inches of any of the company's phone screens based on the screen's width in inches. Show your work.

**SOLUTION** _____

## **Practice** Writing an Equation in Two Variables to Represent a Relationship

➤ **Study the Example showing how to write an equation in two variables to represent a relationship. Then solve problems 1–5.**

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### **Example**

**The total cost of renting a tent is equal to a rental fee plus a cleaning fee of $10. Write an equation with two variables that shows how to find the total cost of renting a tent based on the rental fee.**

You can define variables and use them to write an equation that represents the relationship between the rental fee and the total cost.

$r$ = rental fee in dollars

$t$ = total cost in dollars

The **total cost** is equal to the **rental fee** plus **$10**.

$$t \qquad = \qquad r \qquad + \quad 10$$

The equation $t = r + 10$ can be used to find the total cost of renting a tent.

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**1** Which variable is the independent variable in the Example? Which is the dependent variable? Explain how you know.

**2** The total weight of a delivery truck is equal to the weight of the empty truck plus the weight of the packages. The truck weighs 8,200 lb when empty. Write an equation that shows how to use the total weight of the truck to find the weight of the packages. Show your work.

> **Vocabulary**
>
> **dependent variable**
> a variable whose value depends on the value of a related independent variable.
>
> ⋯⋯⋯⋯⋯⋯⋯⋯⋯⋯
>
> **independent variable**
> a variable whose value is used to find the value of another variable.

**SOLUTION** _____

**3** A scientist is studying the digging behavior of black-footed ferrets.

  **a.** One ferret digs 465 cubic centimeters of soil from a burrow in 5 minutes. Based on this digging rate, write an equation that can be used to predict the amount of soil, *s*, in cubic centimeters the ferret can dig from a burrow in *m* minutes. Show your work.

  **SOLUTION** _____

  **b.** Which variable is the dependent variable in your equation? Explain.

**4** There are 80 Calories in every 4-ounce serving of grapes. Which equation can be used to determine the number of Calories, *y*, in *x* ounces of grapes?

  **A** $y = 20x$

  **B** $y = x + 20$

  **C** $y = 0.05x$

  **D** $y = 84 - x$

**5** A family's dishwasher uses the same number of gallons of water for each load. The table shows how many gallons of water, *g*, the dishwasher uses for *n* loads of dishes. What equation can you use to represent the relationship shown in the table? Show your work.

| Dishwasher Water Use | |
| --- | --- |
| Number of Loads, *n* | Gallons Used, *g* |
| 4 | 18.0 |
| 5 | 22.5 |
| 6 | 27.0 |
| 7 | 31.5 |

**SOLUTION** _____

# Develop Analyzing the Relationship Between Two Variables

➤ **Read and try to solve the problem below.**

A restaurant sells party packs with a combination of samosas and pakoras. Customers ordering online enter the number of samosas they want. The ordering system uses the equation $p = 10 - s$ to determine the number of pakoras, $p$, for a party pack with $s$ samosas. How does the number of pakoras in a party pack change when the number of samosas in the pack increases by 1?

**TRY IT**

✏️ **Math Toolkit** grid paper, two-color counters

**DISCUSS IT**

*Ask:* What did you do first to determine how the number of pakoras changes? Why did you do that step?

*Share:* First, I . . . because . . .

➤ **Explore different ways to analyze the relationship between two variables.**

A restaurant sells party packs with a combination of samosas and pakoras. Customers ordering online enter the number of samosas they want. The ordering system uses the equation $p = 10 - s$ to determine the number of pakoras, $p$, for a party pack with $s$ samosas. How does the number of pakoras in a party pack change when the number of samosas in the pack increases by 1?

## Model It

You can make a table to analyze the relationship between two variables.

| Number of Samosas, $s$ | $10 - s$ | Number of Pakoras, $p$ |
|:---:|:---:|:---:|
| 0 | 10 − 0 | 10 |
| 1 | 10 − 1 | 9 |
| 2 | 10 − 2 | 8 |
| 3 | 10 − 3 | 7 |

## Model It

You can make a graph to analyze the relationship between two variables.

Graph values from the table to show ordered pairs of the form ($s$, $p$).
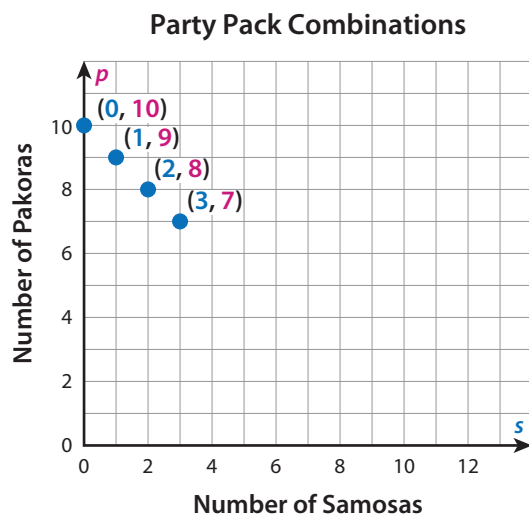
**Party Pack Combinations**

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➤ **Use the problem from the previous page to help you understand how to analyze the relationship between two variables.**

**1** Look at the first **Model It**. Explain why *s* is the independent variable and *p* is the dependent variable in this situation.

**2** Compare each row of the table to the row below it. How does the number of pakoras change when the number of samosas in a party pack increases by 1?

**3** Look at the second **Model It**. How does the graph show the same pattern as the table? Continue this pattern by plotting additional ordered pairs on the graph.

**4** Why can the number of samosas never be greater than 10?

**5** How can a table and a graph help you analyze the relationship between the independent and dependent variables in an equation?

**6** **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to analyze the relationship between two variables.

## Apply It

➤ **Use what you learned to solve these problems.**

**7** A filter cleans the water in Santo's turtle tank. The equation
$y = 2.5x$ models the relationship between the amount of water, $y$,
in gallons that passes through the filter in $x$ minutes.

  **a.** Graph the relationship. Include a title on each axis.

  **b.** How does the graph show the rate at which water passes through
  the filter? How is this rate related to the equation?
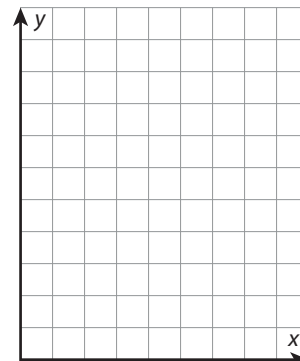
**8** Muna and her father share the same birthday. The equation
$m = f - 32$ relates Muna's age, $m$, to her father's age, $f$, in years.

  **a.** Use the equation to complete the table.

  **b.** How much older is Muna's father than she is? How do the table and
  the equation each show this information?

| Father's Age (yr), $f$ | Muna's Age (yr), $m$ |
|---|---|
| 35 | |
| | 5 |
| | 9 |
| 44 | |

**9** The equation $c = 8t$ can be used to determine the total cost, $c$, in dollars for
$t$ tickets to the school musical.

  **a.** Identify the independent variable and the dependent variable.

  **b.** How does the total cost change when the number of tickets increases by 5?
  Show your work.

**SOLUTION** _____

# Practice Analyzing the Relationship Between Two Variables

➤ **Study the Example showing how to analyze relationships between two variables. Then solve problems 1–4.**

## Example

The equation $d = 3t$ represents the distance, $d$, in kilometers that lava travels in $t$ hours after a volcano erupts. How does the distance the lava travels change for each additional hour that passes?

You can use a table and a graph to analyze the relationship.

The distance the lava travels depends on how much time has passed. So, $t$ is the independent variable, and $d$ is the dependent variable.

| Time (h), $t$ | $3t$ | Distance (km), $d$ |
|---|---|---|
| 0 | 3 • 0 | 0 |
| 1 | 3 • 1 | 3 |
| 2 | 3 • 2 | 6 |
| 3 | 3 • 3 | 9 |
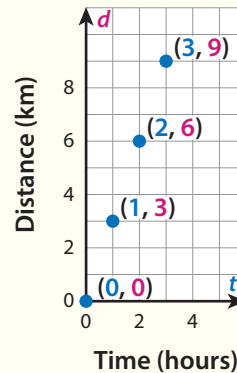
Each time the value of **$t$** increases by 1, the value of **$d$** increases by 3.

With each additional hour, the distance the lava travels increases by 3 km.

**1** Choose one point on the graph in the Example and explain what the point represents in the context of the situation.

**2** The equation $y = x + 8$ represents the length, $y$, in inches of a sleeping bag for a person who is $x$ inches tall. Which ordered pair does not belong on the graph?

**A** (64, 72)

**B** (70, 78)

**C** (72, 80)

**D** (74, 66)

**3** The equation $w = 20n$ models the total weight of honey, $w$, in pounds that a beekeeper expects to collect from $n$ hives.

**a.** Graph the relationship modeled by the equation. Include a title on each axis.

**b.** How does the graph show the expected rate of pounds of honey per hive? How is this rate related to the equation?
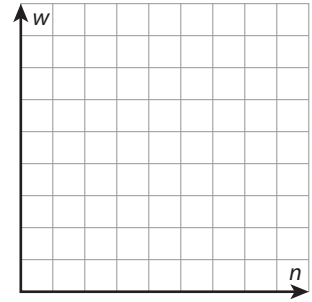


**c.** How many pounds of honey can the beekeeper expect to collect from 6 hives? Show your work.

**SOLUTION** _____

**4** An online company sells watches with different prices. The ordering system uses the equation $c = p + 7.50$ to determine the total cost, $c$, including shipping for a watch with a price of $p$ dollars.

**a.** Identify the independent variable and the dependent variable.

**b.** Use the equation to complete the table.

**c.** How does the total cost change when the price of a watch decreases by \$1? Show your work.

| Price of Watch (\$), $p$ | Total Cost (\$), $c$ |
|---|---|
| 14.00 | |
| | 26.50 |
| 21.00 | |
| | 35.50 |

**SOLUTION** _____

# Refine Analyzing Two-Variable Relationships

➤ **Complete the Example below. Then solve problems 1–8.**

### Example

The equation $c = 6 + 2.5t$ represents the total cost, $c$, to rent skates and skate at a skating rink for $t$ hours. How much does the total cost increase for each hour a person skates?

Look at how you could use a table to analyze the relationship.

| Time (h), $t$ | $6 + 2.5t$ | Total Cost ($), $c$ |
|:---:|:---:|:---:|
| 1 | 6 + 2.5(1) | 8.5 |
| 2 | 6 + 2.5(2) | 11 |
| 3 | 6 + 2.5(3) | 13.5 |
| 4 | 6 + 2.5(4) | 16 |

+1 / +1 / +1     + 2.5 / + 2.5 / + 2.5

**SOLUTION** _____

## Apply It

**1** One trading game card has a mass of 1.71 g. Each pack of trading game cards contains 16 cards. Write an equation with two variables that shows how to find the total mass in grams of the cards in any number of packs of trading game cards. Show your work.

**SOLUTION** _____

**2** The equation $d = 6.5t$ represents the distance, $d$, in meters Conan runs in $t$ seconds. By how much does Conan's distance increase for each additional 30 seconds that he runs? Show your work.

**SOLUTION**

**3** An electronics store offers an accidental damage plan for all laptops. The table shows how the cost of several laptops changes if the damage plan is included. Based on the table, which equation can be used to determine the total cost of a laptop and damage plan based on the laptop's cost without the plan?

| Laptop Cost without Plan ($), $x$ | Laptop Cost with Plan ($), $y$ |
|---|---|
| 296 | 370 |
| 456 | 530 |
| 619 | 693 |
| 779 | 853 |

**A** $y = 0.80x$

**B** $y = 1.25x$

**C** $y = x + 74$

**D** $y = x + 160$

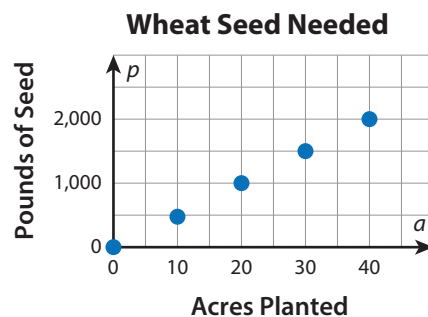Jaime chose B as the correct answer. How might he have gotten that answer?

**4** The graph shows how the amount of wheat seeds a farmer needs depends on the number of acres the farmer is going to plant.

**Wheat Seed Needed**



**a.** Write an equation that shows how to find *p*, the number of pounds of wheat seeds needed to plant *a* acres. Show your work.
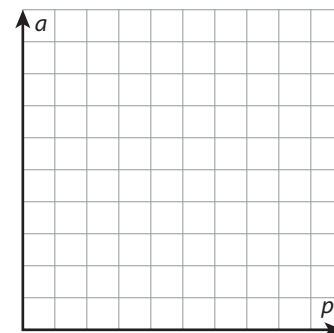
**SOLUTION** _____

**b.** Write an equation that shows the number of acres, *a*, the farmer can plant with *p* pounds of wheat seeds. Explain how you determined your equation.

**Acres That Can Be Planted**



**c.** Graph your equation from problem 4b. How are the two graphs that relate *a* and *p* similar? How are they different?

**5** Workers are preparing an athletic field by mixing soil and sand in the correct ratio. The table shows the volume of sand to mix with different volumes of soil. Which statement is correct?

**A** For 1,425 m³ of soil, the workers should use 375 m³ of sand.

**B** The ratio of the volume of soil to the volume of sand is 1 : 4.

**C** A graph of the relationship includes the point (900, 225).

**D** The equation $y = 4x$ models the relationship.

| Volume of Soil *x*, (m³) | Volume of Sand *y*, (m³) |
|---|---|
| 1,100 | 275 |
| 1,200 | 300 |
| 1,300 | 325 |
| 1,400 | 350 |

**6** A team of scientists is traveling by boat to study a pod, or group, of dolphins. The equation $y = 0.2x$ can be used to find the distance, $y$, in miles the boat travels in $x$ minutes. The dolphins are currently 8.4 mi from the boat. If the dolphins stay where they are, how long will it take the boat to reach them? Show your work.

**SOLUTION** _____

**7** Nadia has a $15 gift card to an online music store. Each song at the store costs $0.99. The equation $m = 15 - 0.99s$ represents the amount of money, $m$, Nadia has left on the gift card after buying $s$ songs.

**a.** The independent variable is _____ and the dependent variable is _____ .

**b.** How does the amount of money left on the card change for each extra song Nadia buys? Show your work.

**SOLUTION** _____

**8** **Math Journal** Write a word problem that can be solved using an equation in two variables. Explain how to solve your problem.

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✅ **End of Lesson Checklist**

☐ **INTERACTIVE GLOSSARY** Find the entry for *dependent variable*. Tell how dependent variables and independent variables are different.

☐ **SELF CHECK** Go back to the Unit 5 Opener and see what you can check off.