LESSON 6

Dear Family,

This week your student is learning about the greatest common factor and the least common multiple of two whole numbers.

The **greatest common factor (GCF)** of two numbers is the greatest factor the two numbers have in common. For example, the GCF of 12 and 18 is 6.

Factors of 12: **1**, **2**, **3**, 4, **6**, 12

Factors of 18: **1**, **2**, **3**, **6**, 9, 18

The **least common multiple (LCM)** of two numbers is the least multiple that the two numbers share. For example, the LCM of 12 and 18 is 36.

Multiples of 12: 12, 24, **36**, 48, . . .

Multiples of 18: 18, **36**, 54, . . .

Your student will be learning to use the greatest common factor to solve problems like the one below.

> A teacher gives out 30 markers and 40 colored pencils. He gives an equal number of markers and an equal number of colored pencils to each student. What is the greatest number of students the teacher can give markers and colored pencils to?

➤ **ONE WAY** to find the greatest common factor of two whole numbers is to list all the factors of each number.

Factors of 30: **1**, **2**, 3, **5**, 6, (**10**), 15, 30

Factors of 40: **1**, **2**, 4, **5**, 8, (**10**), 40

The common factors of 30 and 40 are 1, 2, 5, and 10. The greatest common factor is 10.

➤ **ANOTHER WAY** is to write each number as a product of prime factors. Then multiply the common prime factors.

$30 = 3 \cdot 2 \cdot 5$ $\qquad$ $40 = 2 \cdot 2 \cdot 2 \cdot 5$

Greatest common factor: $2 \cdot 5 = 10$

Using either method, the greatest number of students the teacher can give an equal number of markers and an equal number of colored pencils to is 10.

Use the next page to start a conversation about factors.

Activity Exploring Common Factors

Do this activity together to explore common factors.

You are helping with the 6th grade spirit day at school. There will be games with prize bags for winners. Your teacher asks you to use these supplies to make prize bags. All the prize bags must contain the same items.

Different combinations of supplies will result in different numbers of prize bags. Which combination would you choose and why?

42 puzzles
48 bouncy balls
30 sets of paint
36 markers

COMBINATION 1

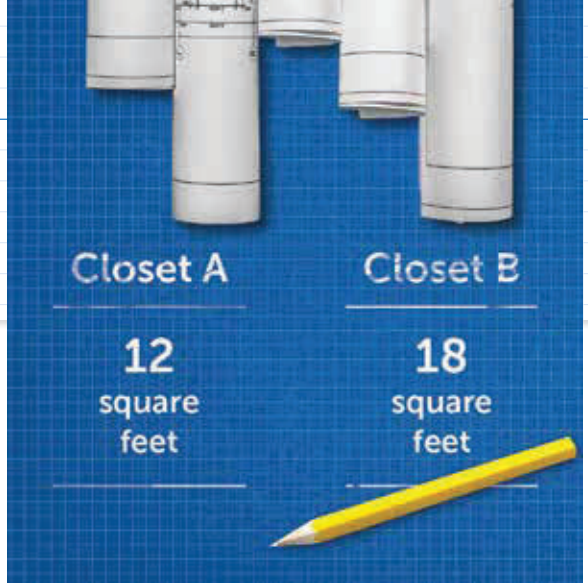
30 prize bags Items in each bag	Leftover supplies
1 puzzle	12 puzzles
1 bouncy ball	18 bouncy balls
1 set of paint	6 markers
1 marker	

COMBINATION 2

15 prize bags Items in each bag	Leftover supplies
2 puzzles	12 puzzles
3 bouncy balls	3 bouncy balls
2 sets of paint	6 markers
2 markers	

What combination of prizes would result in zero leftover supplies?

Explore Common Factors and Multiples



Previously, you learned how to find factors and multiples of a whole number. In this lesson, you will learn how to find the greatest common factor and the least common multiple of two whole numbers.

➤ **Use what you know to try to solve the problem below.**

A floor plan for a new house shows a linen closet with an area of 12 ft^2 and a clothes closet with an area of 18 ft^2. The two closets will share one wall. What are all the possible whole-number lengths for the shared wall?

TRY IT

Math Toolkit counters, grid paper, sticky notes, unit tiles

DISCUSS IT

Ask: How would you explain what the problem is asking in your own words?

Share: The problem is asking . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7, SMP 8

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

CONNECT IT

1 **Look Back** Is there more than one possible side length for the wall shared by the two closets? Explain how you know.

2 **Look Ahead** The length of the shared wall is a common factor of 12 and 18 because it divides both 12 and 18 without a remainder. You can use common factors in situations that involve dividing objects into equal groups.

Suppose Malik wants to share a bag of connecting cubes with some friends. He has 6 blue cubes and 15 red cubes. He wants each friend to get the same number of blue cubes and the same number of red cubes.

a. List all the ways Malik can put the blue cubes into equal groups.

b. List all the ways Malik can put the red cubes into equal groups.

c. Malik wants to share all the cubes. How many friends can Malik give cubes to? Explain how you know.

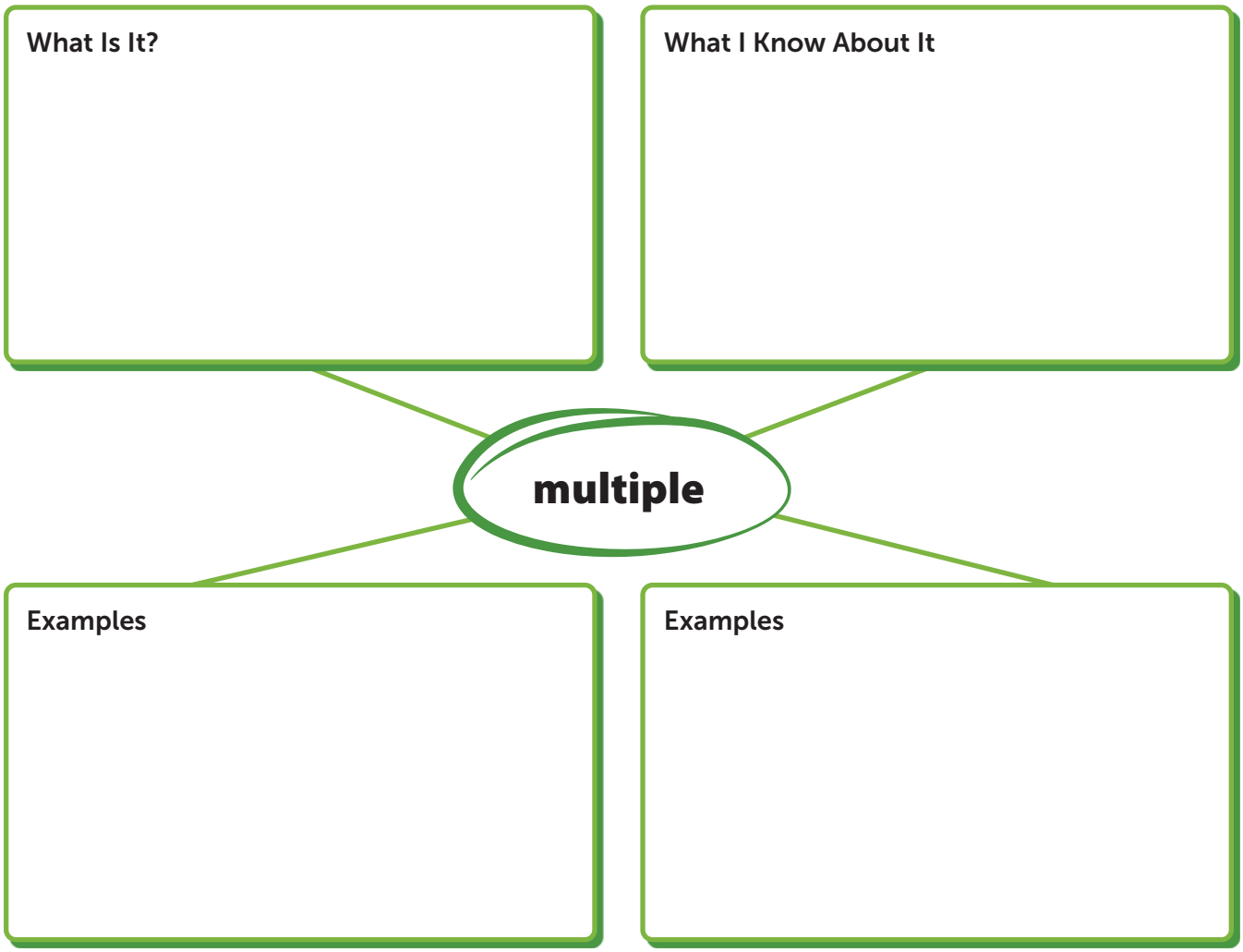
d. How is your answer to problem 2c related to factors of the numbers 6 and 15?

3 **Reflect** Hugo says that some whole numbers have no common factors. Jasmine says any two whole numbers will always have at least one common factor. Who is correct? Explain.

 Name:

Prepare for Finding Greatest Common Factor and Least Common Multiple

1 Think about what you know about factors and multiples. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Any multiple of 12 is also a multiple of 4. Explain why this is true.

3 Rectangular paintings hang on the wall of an art gallery. One painting has an area of 24 ft^2. Another painting has an area of 32 ft^2. The paintings have whole-number side lengths and have one pair of side lengths in common.

a. What could the common side length be? Show your work.

SOLUTION _____

b. Check your answer to problem 3a. Show your work.

Develop Finding the Greatest Common Factor of Two Whole Numbers

➤ **Read and try to solve the problem below.**

Akio has 27 blankets and 18 flashlights to use in emergency relief kits for communities in need. He will put the same number of blankets and the same number of flashlights in each kit. He wants to use all the blankets and flashlights. What is the greatest number of emergency relief kits Akio can make?

TRY IT

Math Toolkit counters, multiplication tables, number lines, unit tiles

DISCUSS IT

Ask: How does your strategy show the greatest number of kits?

Share: In my solution, . . . represents . . .

➤ **Explore different ways to find the greatest common factor of two whole numbers.**

Akio has 27 blankets and 18 flashlights to use in emergency relief kits for communities in need. He will put the same number of blankets and the same number of flashlights in each kit. He wants to use all the blankets and flashlights. What is the greatest number of kits Akio can make?

Analyze It

You can list all the factors of each number to find the factors that divide both numbers without a remainder.

Factors of 27: 1, 3, 9, 27

Factors of 18: 1, 2, 3, 6, 9, 18

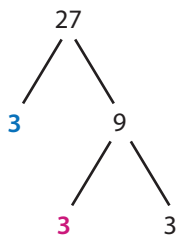
The common factors of 27 and 18 are 1, 3, and 9.

The **greatest common factor (GCF)** of 27 and 18 is 9.

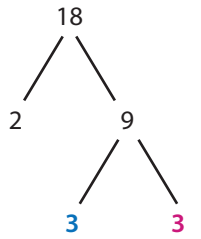
Model It

You can use the prime factors of two numbers to find their greatest common factor.

One way to show all the prime factors of a number is with a factor tree.



$27 = 3 \times 3 \times 3$



$18 = 2 \times 3 \times 3$

The numbers 27 and 18 share two factors of 3.

$GCF = 3 \times 3$

$= 9$

CONNECT IT

Use the problem from the previous page to help you understand how to find the greatest common factor (GCF) of two whole numbers.

1. Look at **Analyze It**. How are the factors of 27 and 18 related to the number of kits Akio can make?

2. Look at **Analyze It** and **Model It**. They both show that 9 is the greatest common factor (GCF) of 27 and 18. What does the GCF represent in this situation?

3. How many blankets and how many flashlights will be in each kit if Akio uses the GCF as the number of kits? Where do you see these amounts in the equations under the factor trees?

4. Jada writes the equations under the factor trees using exponents.

   $27 = 3^3$ and $18 = 2 \times 3^2$

   How could using exponents help you find the GCF of 18 and 27?

5. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the greatest common factor of two whole numbers.

Apply It

Use what you learned to solve these problems.

6 Find the greatest common factor (GCF) of 56 and 96. Show your work.

SOLUTION _____

7 Reth says the greatest common factor of 36 and 60 is 6. Is Reth correct? Explain.

8 Luis buys 16 bellflowers and 20 roses. He wants to plant an equal number of bellflowers and an equal number of roses in each of his flower boxes. What is the greatest number of flower boxes Luis can plant using all the bellflowers and roses? Show your work.

SOLUTION _____

Practice Finding the Greatest Common Factor of Two Whole Numbers

➤ **Study the Example showing how to find the greatest common factor of two whole numbers. Then solve problems 1–4.**

Example

Kennedy plans to completely cover the wall behind her kitchen sink with square tiles. The wall is a rectangle that is 15 in. high and 24 in. long. Each square tile will have whole-number side lengths. What is the side length of the largest square tile that Kennedy can use?

The side length of the tile must divide both 15 in. and 24 in.

List the factors of 15: **1**, **3**, 5, 15

List the factors of 24: **1**, 2, **3**, 4, 6, 8, 12, 24

The only whole numbers that divide into both 15 and 24 are **1** and **3**.
The greatest common factor of 15 and 24 is 3.

The side length of the largest tile Kennedy can use is 3 in.

1 Suppose the wall in the Example is 16 in. high and 28 in. long.

a. What is the side length of the largest square tile that Kennedy can use? Show your work.

SOLUTION _____

b. How many tiles does Kennedy need to completely fill the wall? Show your work.

SOLUTION _____

2 Avery says the greatest common factor of 12 and 15 is 60. Is Avery correct? Explain how you know.

3 What is the GCF of 60 and 100? Show your work.

SOLUTION

4 An art teacher is making packages of paintbrushes and paint for his students. He has 24 brushes and 40 tubes of paint. Each package will have the same number of brushes and the same number of tubes of paint.

a. What is the greatest number of packages that the art teacher can make using all the paintbrushes and paint? Show your work.

SOLUTION

b. How many paintbrushes and tubes of paint are in each package?

Develop Finding the Least Common Multiple of Two Whole Numbers

➤ **Read and try to solve the problem below.**

Morgan and Anne own North Country Sheep Farm. Morgan shears one herd of sheep every 6 months. Anne decides to shear another herd of sheep every 8 months to see if waiting longer results in higher quality wool.

Both herds are sheared this month. In how many months will Morgan and Anne shear their herds in the same month again?

TRY IT

Math Toolkit counters, grid paper, hundred chart, number lines

DISCUSS IT

Ask: How does your strategy show the number of months before Morgan and Anne shear their herds in the same month again?

Share: In my solution, . . . represents . . .

➤ **Explore different ways to find the least common multiple of two whole numbers.**

Morgan and Anne own North Country Sheep Farm. Morgan shears one herd of sheep every 6 months. Anne decides to shear another herd of sheep every 8 months to see if waiting longer results in higher quality wool.

Both herds are sheared this month. In how many months will Morgan and Anne shear their herds in the same month again?

Model It

You can list multiples of 6 and 8 to show the numbers of months until the sheep from each herd will have their wool sheared.

Multiples of 6: 6, 12, 18, **24**, 30, 36, 42, **48**, . . .

Multiples of 8: 8, 16, **24**, 32, 40, **48**, 56, 64, . . .

The numbers **24** and **48** that appear in both lists are common multiples of 6 and 8.

The **least common multiple (LCM)** of 6 and 8 is 24.

Model It

You can use the prime factors of two numbers to find their least common multiple.

$6 = 2 \times 3$ $\qquad$ $8 = 2 \times 2 \times 2$

6 and 8 have one common factor of **2**.

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

Multiply the factors of the numbers. Use the common factor only once.

$\text{LCM} = 2 \times 2 \times 2 \times 3$

$= 24$

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the least common multiple of two whole numbers.**

1. The numbers 24 and 48 are common multiples of 6 and 8. What do these numbers represent in this situation?

2. Both **Model Its** show that the least common multiple (LCM) of 6 and 8 is 24. What does the LCM represent in this situation?

3. Why is the common factor of 2 used only once in the LCM of 6 and 8?

4. The product of 6 and 8 is a common multiple of 6 and 8, but it is not the least common multiple of 6 and 8. Look at these statements about least common multiples. Underline the LCMs that are the product of the two numbers.

   The LCM of 4 and 5 is 20. The LCM of 4 and 6 is 12.

   The LCM of 4 and 3 is 12. The LCM of 4 and 8 is 8.

   When can you multiply two numbers to find their LCM?

5. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the least common multiple of two whole numbers.

Apply It

Use what you learned to solve these problems.

6 Ria is buying plates and cups for a party. She wants the same number of each. Plates are sold in packs of 8. Cups are sold in packs of 12. What is the least number of plates and cups that Ria can buy?

A 12

B 24

C 48

D 96

7 In problem 6, suppose that Ria wants at least 40 plates and 40 cups. Could Ria buy exactly 40 plates and 40 cups? Explain.

8 When Carlos goes to his favorite restaurant for lunch, he gets a sandwich and a smoothie. The restaurant has two rewards programs.

- Buy 10 sandwiches, get one free sandwich.
- Buy 4 smoothies, get one free smoothie.

How many times does Carlos need to buy a sandwich and a smoothie to get both of them for free on the same visit? Show your work.

SOLUTION

Practice Finding the Least Common Multiple of Two Whole Numbers

➤ **Study the Example showing how to find the least common multiple of two whole numbers. Then solve problems 1–5.**

Example

At a train station, trains stop on track A and track B at 8:00 AM. A train stops on track A every 9 minutes. A train stops on track B every 12 minutes. When will trains stop on both tracks at the same time again?

Find the least common multiple (LCM) of 9 minutes and 12 minutes.

Multiples of 9: 9, 18, 27, **36**, 45, 54, **60**, . . .

Multiples of 12: 12, 24, **36**, 48, **60**, 72, . . .

The numbers **36** and **60** are common multiples of 9 and 12. The LCM is 36.

It will be 36 minutes until a train stops on each track at the same time again. So, trains will stop on both tracks again at 8:36 AM.

1 A train stops on track C every 10 minutes. A train stops on track D every 6 minutes. Trains stop on track C and track D at 10:30 AM. When will trains stop on both tracks at the same time again? Show your work.

SOLUTION _____

2 Find the LCM of 6 and 9. Show your work.

SOLUTION _____

3 What is the least common multiple (LCM) of 7 and 10? Show your work.

SOLUTION _____

4 The LCM of two numbers is 18. One of the numbers is 9. The other number is less than 9. What could the other number be? Show your work.

SOLUTION _____

5 Pilar swims every 4 days and jogs every 6 days. She did both activities today. How many days from now will she both swim and jog again? Show your work.

SOLUTION _____

Refine Finding Greatest Common Factor and Least Common Multiple

➤ **Complete the Example below. Then solve problems 1–9.**

Example

What is the least common multiple (LCM) of 5, 8, and 10?

Look at how you could show your work using lists of multiples.

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, (40), 45

Multiples of 8: 8, 16, 24, 32, (40), 48, 56, 64, 72

Multiples of 10: 10, 20, 30, (40), 50, 60, 70, 80, 90

The first number to appear in all three lists is 40.

SOLUTION _____

CONSIDER THIS . . .
You can use the same strategies to find the LCM of three numbers that you use to find the LCM of two numbers.

PAIR/SHARE
If you continue listing multiples of 5, 8, and 10, what will be the next number to appear in all three lists?

Apply It

1 Solve the puzzle from the clues. Show your work.

Clue 1: We are two whole numbers less than or equal to 12.

Clue 2: Our least common multiple is 36.

Clue 3: Our greatest common factor is 1.

What two whole numbers are we?

SOLUTION _____

CONSIDER THIS . . .
If the GCF of two numbers is 1, it means they share no other common factors.

PAIR/SHARE
Which clue did you start with? Why?

2 Chantel has 45 green balloons and 54 purple balloons to make into bunches for a school celebration. She wants each bunch to have the same number of each color balloon. What is the greatest number of bunches Chantel can make if she wants to use all of her balloons? How many purple balloons will she put in each bunch? Show your work.

CONSIDER THIS . . .
Do you need to find the greatest common factor or the least common multiple?

SOLUTION

PAIR/SHARE
Would the number of bunches be different if Chantal had 27 purple balloons instead of 54?

3 What is the least common multiple of 5 and 10?

A 1

B 5

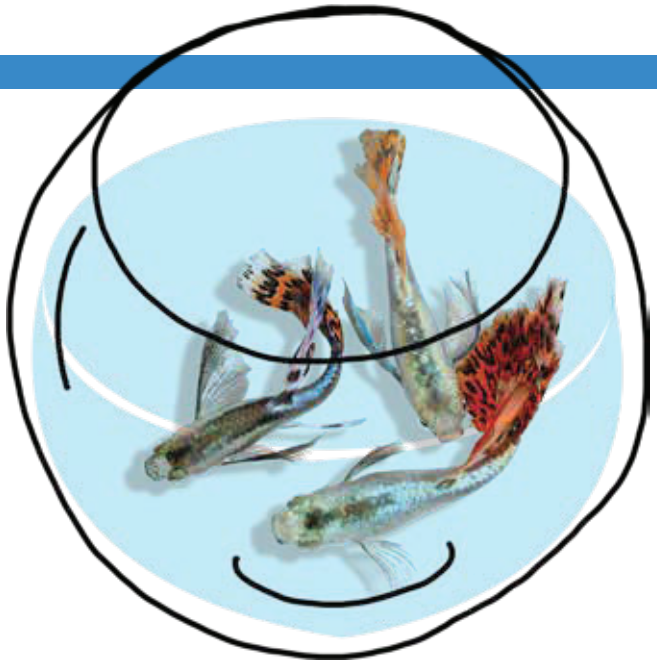
C 10

D 50

Alyssa chose B as the correct answer. How might she have gotten that answer?

CONSIDER THIS . . .
Can the least common multiple of two numbers be less than one of the numbers?

PAIR/SHARE
When is the product of two numbers also the LCM of the two numbers?



4 Ignacio buys three types of fish. He buys 12 guppies, 9 mollies, and 15 swordtails. He plans to divide the fish between more than one fish tank. He wants to put the same number of each type of fish into each tank. How many tanks should Ignacio use? How many of each type of fish will be in each tank? Show your work.

SOLUTION _____

_____

5 The GCF of 20 and another number is 4. Which of these could be the other number? Select all that apply.

A 4

B 8

C 10

D 20

E 80

6 Inés finds the greatest common factor and the least common multiple of 10 and 12. She then subtracts to find the difference between the GCF and the LCM. What is the difference? Show your work.

SOLUTION _____

7 Tell whether each statement about 20 and 30 is *True* or *False*.

	True	False
a. The greatest common factor is 5.	○	○
b. 10 is a common multiple.	○	○
c. The least common multiple is 60.	○	○
d. 2 is a common factor.	○	○

8 **a.** Find the GCF of 8 and 12. Show that $\frac{8}{12} = \frac{2}{3}$ by dividing the numerator and denominator of $\frac{8}{12}$ by the GCF.

b. Show how to add the fractions $\frac{1}{8}$ and $\frac{1}{12}$ by using the LCM of 8 and 12 as the common denominator.

9 **Math Journal** Write two different whole numbers that have 6 as their greatest common factor. Explain how you found your two numbers. Then find the least common multiple of your numbers.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entries for *greatest common factor* and *least common multiple*. Tell how greatest common factor and least common multiple are different.

☐ **SELF CHECK** Go back to the Unit 1 Opener and see what you can check off.