

# Understand Multiplication by a Fraction



Dear Family,

This week your child is exploring multiplying fractions.

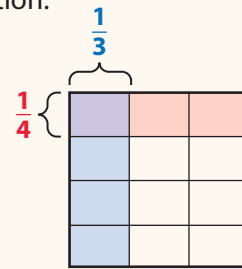
An area model can help you visualize finding a fraction of a fraction.

The model shows  $\frac{1}{4}$  and  $\frac{1}{3}$  of the same whole.

Each row shows  $\frac{1}{4}$  of the whole.

Each column shows  $\frac{1}{3}$  of the whole.

The part shaded purple shows  $\frac{1}{4}$  of  $\frac{1}{3}$  of the whole, or  $\frac{1}{12}$ .



Your child is learning that finding a fraction of a fraction is the same as finding the product of the fractions. Your child might see a problem like the one below.

*If  $\frac{2}{3}$  of the gym floor has been cleaned and students can play on  $\frac{3}{4}$  of the cleaned floor, what part of the whole gym floor can the students play on?*

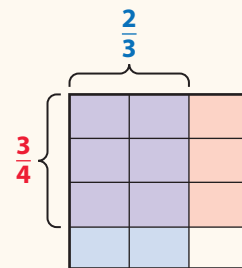
To solve the problem, you find  $\frac{3}{4}$  of  $\frac{2}{3}$ , or  $\frac{3}{4} \times \frac{2}{3}$ .

The model shows  $\frac{3}{4}$  and  $\frac{2}{3}$  of the same whole.

3 rows show  $\frac{3}{4}$  of the whole.

2 columns show  $\frac{2}{3}$  of the whole.

The part shaded purple shows  $\frac{3}{4}$  of  $\frac{2}{3}$  of the whole.



The model is divided into 12 equal parts, 6 of which are shaded purple.

You can see that  $\frac{6}{12}$  of the whole is shaded purple. So,  $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ .

Students can play on  $\frac{6}{12}$ , or  $\frac{1}{2}$ , of the gym floor.

Invite your child to share what he or she knows about multiplying fractions by doing the following activity together.



## ACTIVITY MULTIPLY BY A FRACTION

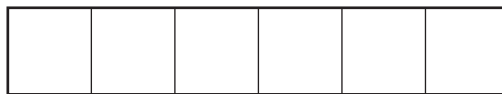
Do this activity with your child to understand multiplication by a fraction.

**Materials** 2 different colors of crayons or colored pencils, number cube



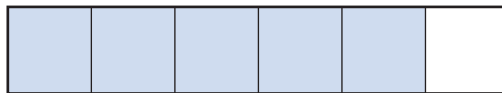
- Together with your child, draw a blank rectangle at the bottom of the page to show the product of two fractions.
- One person rolls the number cube. This number tells how many equal parts to show in the rectangle. Draw vertical lines to show the equal parts.

*Example:* Roll a 6 and draw vertical lines to show 6 equal parts in the rectangle.



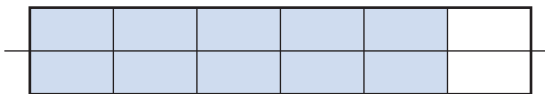
- The same person shades a fraction of the rectangle and names that fraction.

*Example:* Shade  $\frac{5}{6}$ .



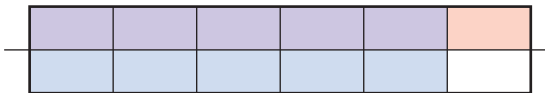
- The other person rolls the number cube. This number tells how many equal parts to show in the same rectangle. Draw horizontal lines to show the equal parts.

*Example:* Roll a 2 and draw a horizontal line to show 2 equal parts (top and bottom) of the rectangle.



- The same person shades a fraction of the rectangle and names that fraction.

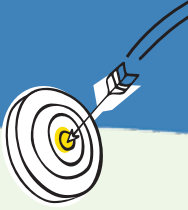
*Example:* Shade  $\frac{1}{2}$ .



- The part where the shading overlaps shows the product. Together, write the fraction multiplication equation that the picture shows.

*Example:*  $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

# Explore Multiplication by a Fraction



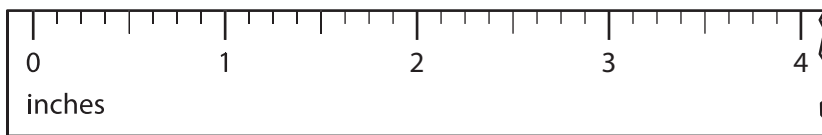
What does it mean to multiply by a fraction?



## MODEL IT

Complete the models below.

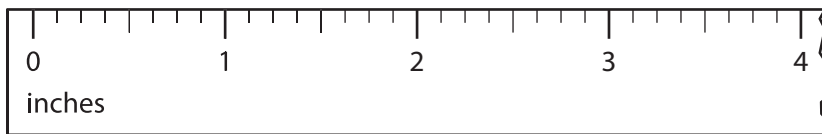
- Shade and label three  $\frac{1}{2}$ -inch sections of the ruler. Then complete the sentence and the multiplication equation that represents the total length you shaded.



3 sections of  $\frac{1}{2}$  inch are ..... inches.

$$3 \times \frac{1}{2} = \dots\dots\dots$$

- Shade and label  $\frac{1}{2}$  of a 3-inch section of the ruler. Then complete the sentence and the multiplication equation that represents the total length you shaded.



$\frac{1}{2}$  of 3 inches is ..... inches.

$$\frac{1}{2} \times 3 = \dots\dots\dots$$

## Learning Targets

- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
- Interpret the product  $\frac{a}{b} \times q$  as  $a$  parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ .

**SMP** 1, 2, 3, 4, 5, 6, 8



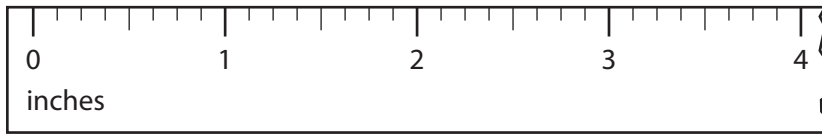
## DISCUSS IT

- How does the length of three  $\frac{1}{2}$ -inch segments compare to the length of half of a 3-inch segment?
- I think  $\frac{1}{2}$  of 3 is equal to both  $3 \div 2$  and  $3 \times \frac{1}{2}$  because ...

# MODEL IT

Complete the models below.

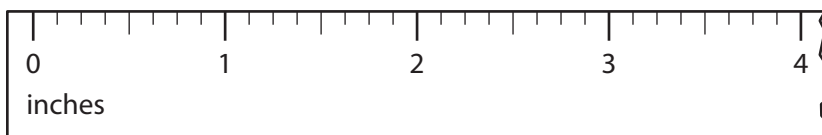
- 3 Shade and label the ruler to find the length of  $\frac{1}{4}$  of  $\frac{1}{2}$  inch. Complete the sentence and the multiplication equation that represents the total length you shaded.



$\frac{1}{4}$  of  $\frac{1}{2}$  inch is ..... of an inch.

$\frac{1}{4} \times \frac{1}{2} =$  .....

- 4 Shade and label the ruler to show  $\frac{3}{4}$  of  $\frac{1}{2}$  inch. Write a multiplication equation that represents the total length you shaded.



Equation .....

## 5 REFLECT

Based on the two problems above, explain how to find  $\frac{2}{4}$  of  $\frac{1}{2}$ , or  $\frac{2}{4} \times \frac{1}{2}$ .

.....

.....

.....



## DISCUSS IT

- How many parts did you divide the whole inch into when you found  $\frac{1}{4}$  of  $\frac{1}{2}$  inch?
- I think finding  $\frac{1}{4}$  of a number or multiplying a number by  $\frac{1}{4}$  is like dividing by 4 because ...

# Prepare for Multiplication by a Fraction

- 1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

<p>In My Own Words</p>	<p>My Illustrations</p>
<p>Examples</p>	<p>Non-Examples</p>

**equal parts**

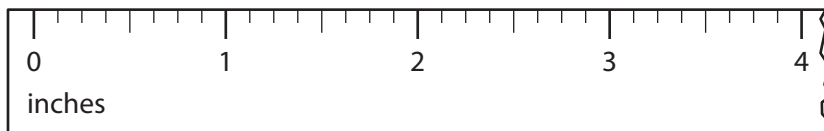
- 2 Shade and label three  $\frac{1}{4}$ -inch sections of the ruler. Complete the sentence and the multiplication equation that represents the total length you shaded.



3 sections of  $\frac{1}{4}$  inch are ..... inch.       $3 \times \frac{1}{4} = \dots\dots\dots$

Solve.

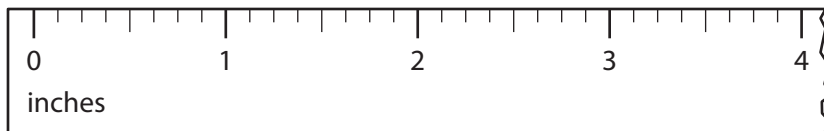
- 3 Shade and label the ruler to find the length of  $\frac{1}{2}$  of  $\frac{1}{2}$  inch. Complete the sentence and the multiplication equation that represents the total length you shaded.



$\frac{1}{2}$  of  $\frac{1}{2}$  inch is ..... inch.

$\frac{1}{2} \times \frac{1}{2} = \dots\dots\dots$

- 4 Shade and label the ruler to show  $\frac{1}{2}$  of  $\frac{3}{4}$  inch. Write a multiplication equation that represents the total length you shaded.



**Equation** .....

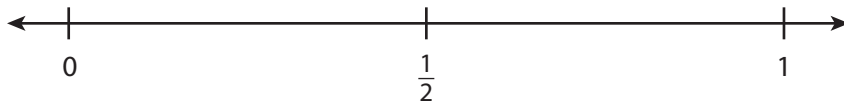


# Develop Understanding of Multiplying by a Fraction

## MODEL IT: NUMBER LINES

Try these two problems.

- 1 Mark and shade the number line to show  $\frac{2}{5}$  of  $\frac{1}{2}$ .

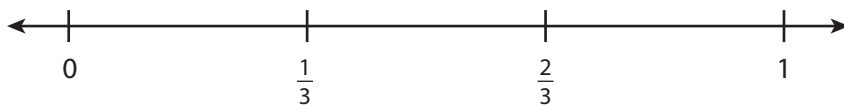


What fraction of the whole did you shade? .....

Complete the multiplication equation.

$\frac{2}{5} \times \frac{1}{2} =$  .....

- 2 Mark and shade the number line to show  $\frac{3}{4}$  of  $\frac{1}{3}$ .  
Then write a multiplication equation for the model.



Equation .....



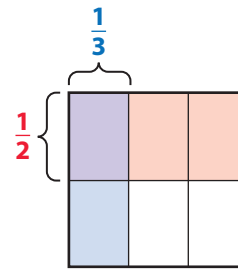
### DISCUSS IT

- How did you know how many of the parts to shade for each problem?
- I think the number line in problem 1 shows  $\frac{2}{5} \times \frac{1}{2}$  because ...

## MODEL IT: AREA MODELS

Use the area models to show multiplying fractions.

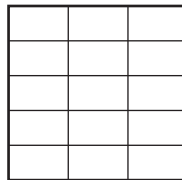
- 3 Explain why the purple rectangle, where the shading overlaps, represents  $\frac{1}{2}$  of  $\frac{1}{3}$ . Then complete the product.



$$\frac{1}{2} \times \frac{1}{3} = \dots\dots\dots$$

- 4 Shade the area model to show  $\frac{2}{5}$  of  $\frac{2}{3}$ . Then complete the product.

$$\frac{2}{5} \times \frac{2}{3} = \dots\dots\dots$$



### DISCUSS IT

- Were you and your partner's explanations in problem 3 the same?
- I think the area model in problem 4 shows  $\frac{2}{5} \times \frac{2}{3}$  because . . .

## CONNECT IT

Complete the problems below.

- 5 How are the number line and area models for fraction multiplication alike? How are they different?

- 6 Choose any model you like to show  $\frac{1}{3} \times \frac{3}{4}$ .

$$\frac{1}{3} \times \frac{3}{4} = \dots\dots\dots$$



# Practice Multiplying by a Fraction

Study how the Example shows multiplying a fraction by a fraction.  
Then solve problems 1–6.

## EXAMPLE

Use an area model to find the product  $\frac{2}{3} \times \frac{3}{5}$ .

You can think of the product  $\frac{2}{3} \times \frac{3}{5}$  as finding  $\frac{2}{3}$  of  $\frac{3}{5}$ .

Each row is  $\frac{1}{3}$  of the whole.

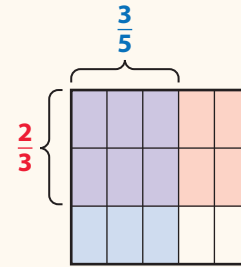
Each column is  $\frac{1}{5}$  of the whole.

The whole is divided into 15 equal parts.

The purple parts show  $\frac{2}{3}$  of  $\frac{3}{5}$ .

Because 6 out of 15 parts of the whole are shaded purple, the purple part shows  $\frac{6}{15}$ .

$$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$$



1 Why are fifteenths shown in the Example model?

2 Use the area model in the Example to write each product.

$\frac{1}{3} \times \frac{3}{5} = \dots\dots\dots$

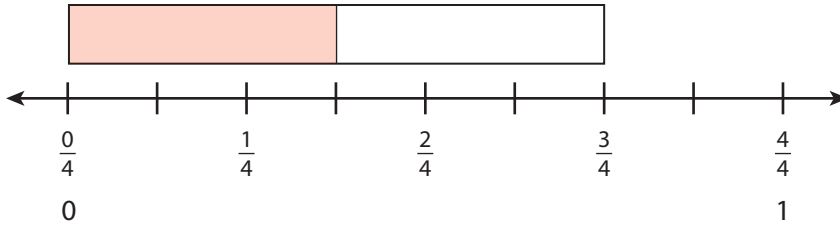
$\frac{2}{3} \times \frac{4}{5} = \dots\dots\dots$

$\frac{3}{3} \times \frac{2}{5} = \dots\dots\dots$

3 Choose Yes or No to tell whether the denominator of each product is 12.

	Yes	No
$\frac{1}{2} \times \frac{1}{6}$	(A)	(B)
$\frac{3}{4} \times \frac{2}{5}$	(C)	(D)
$\frac{1}{4} \times \frac{2}{3}$	(E)	(F)
$\frac{5}{6} \times \frac{2}{2}$	(G)	(H)

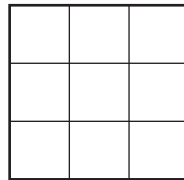
4 The number line shows  $\frac{1}{2} \times \frac{3}{4}$ .



- a. Each fourth on the number line is divided into how many equal parts? .....
- b. Each of these parts is what fraction of the whole? .....
- c. How many eighths of the whole are shaded? .....
- d.  $\frac{1}{2} \times \frac{3}{4} =$  .....

5 Use the area model to help you find each product.

- a.  $\frac{1}{3} \times \frac{1}{3} =$  .....
- b.  $\frac{1}{3} \times \frac{2}{3} =$  .....
- c.  $\frac{2}{3} \times \frac{2}{3} =$  .....
- d.  $\frac{3}{3} \times \frac{2}{3} =$  .....
- e.  $\frac{3}{3} \times \frac{3}{3} =$  .....



6 Tell whether each statement is *True* or *False* for the product  $\frac{2}{4} \times \frac{3}{5}$ .

	True	False
The denominator of the product is 20.	(A)	(B)
The denominator of the product is 9.	(C)	(D)
The product is less than each factor.	(E)	(F)
The product is greater than each factor.	(G)	(H)

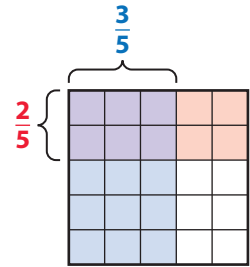
# Refine Ideas About Multiplying by a Fraction

## APPLY IT

Complete these problems on your own.

### 1 DESCRIBE

Tell what multiplication problem the model shows. Explain why.



### 2 EXPLAIN

Landon said that  $\frac{2}{3} \times \frac{1}{6} = \frac{5}{6}$ . Tell how Landon found his product and then explain how to find the correct product.

### 3 ANALYZE

Fill in the products in the table. Look for patterns as you work.

$\frac{1}{4} \times \frac{1}{3} =$	$\frac{2}{4} \times \frac{1}{3} =$	$\frac{3}{4} \times \frac{1}{3} =$	$\frac{4}{4} \times \frac{1}{3} =$
$\frac{1}{4} \times \frac{2}{3} =$	$\frac{2}{4} \times \frac{2}{3} =$	$\frac{3}{4} \times \frac{2}{3} =$	$\frac{4}{4} \times \frac{2}{3} =$

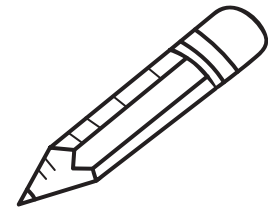
How do the numerators and denominators of two fractions relate to the numerator and denominator of their product?

### PAIR/SHARE

Discuss your solutions for these three problems with a partner.

Use what you have learned to complete problem 4.

- 4 **Part A** Start with a sheet of paper. Fold it in half and then fold it in half again. Continue folding the paper in half. After each fold, fill in the blanks to tell what fraction of the whole each equal part is.



$$\frac{1}{2} \times 1 = \dots\dots\dots$$

$$\frac{1}{2} \times \frac{1}{2} = \dots\dots\dots$$

$$\frac{1}{2} \times \dots\dots\dots = \dots\dots\dots$$

$$\frac{1}{2} \times \dots\dots\dots = \dots\dots\dots$$



**Part B** Look at the denominators of the products you found in Part A. Each time you multiply by  $\frac{1}{2}$ , what happens to the denominator?

Without folding the paper, what do you think the next product in the pattern will be? Why?

**Part C** Now look at the sizes of the equal parts of the folded paper. Each time you fold the paper in half, what happens to the size of the parts?

5 **MATH JOURNAL**

Genet says that  $\frac{3}{8}$  of  $\frac{1}{2}$  is  $\frac{3}{16}$  because  $\frac{1}{8}$  of  $\frac{1}{2}$  is  $\frac{1}{16}$ . Is Genet correct? Explain.