Multiply Fractions to Find Area

LESSON 20

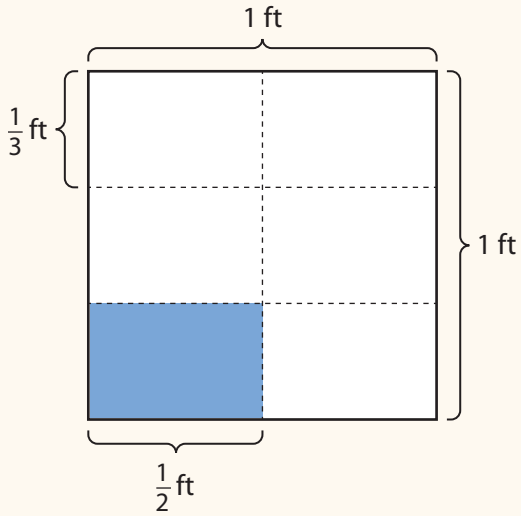
Dear Family,

This week your child is learning to multiply fractions to find the area of rectangles.

He or she might see a problem like this:

> *Mark has a square placemat that measures 1 foot on each side. He divides it in half vertically and in thirds horizontally. He wants to decorate each part with a different pattern. What is the area of each part of the placemat?*

To understand the problem, your child could draw and label a picture.



The dashed lines show 6 equal parts.

Each part is $\frac{1}{2}$ foot wide and $\frac{1}{3}$ foot long. Each part is $\frac{1}{6}$ of the whole.

Multiply to find the area of each part.

$$\frac{1}{2} \text{ foot} \times \frac{1}{3} \text{ foot} = \frac{1}{6} \text{ square foot}$$

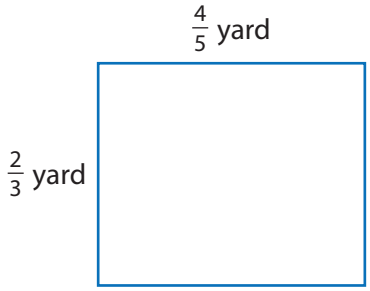
The area of each part of the placemat is $\frac{1}{6}$ square foot.

Invite your child to share what he or she knows about multiplying fractions to find the area of rectangles by doing the following activity together.

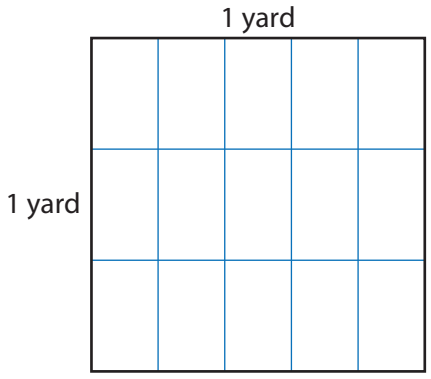
ACTIVITY MULTIPLYING FRACTIONS TO FIND AREA

Do this activity with your child to find the area of a rectangle by multiplying fractions.

- Look at the rectangle below.



- Remind your child that you can find the area of a rectangle by multiplying the length by the width. (area = length × width)
- Together with your child, find the area of the rectangle shown above by multiplying the length by the width.
- Check your answer by using an area model. The square below has an area of 1 square yard. Ask your child to shade parts of the square below to show the same area as the rectangle above.



- Together with your child, find the area of the shaded part of the square by finding the fraction of the square that is shaded. Ask your child: *Does this match your answer from above?*

Answer: Area = $\frac{4}{5}$ yard × $\frac{2}{3}$ yard = $\frac{8}{15}$ square yard

Explore Multiplying Fractions to Find Area

Previously, you learned about multiplying fractions. Now you will use area models to multiply fractions and find areas of rectangles. Use what you know to try to solve the problem below.

Mr. Thompson designs a 1-mile square park. He makes a square with $\frac{5}{10}$-mile sides on his design for a dog play space. How many square miles of the park does he use for the dog play space?

Learning Target

- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Math Toolkit

- fraction models
- multiplication models
- geoboard
- rubber bands
- base-ten blocks
- grid paper
- index cards

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I am not sure how to find the answer because . . .

CONNECT IT

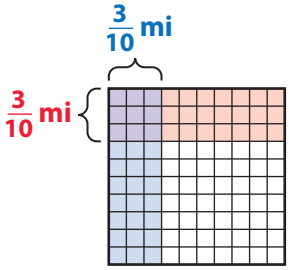
1 LOOK BACK

Explain how you found the area of the park used for the dog play space.

2 LOOK AHEAD

Suppose you want to find the area of a rectangular section of the park that is $\frac{3}{10}$ mile by $\frac{3}{10}$ mile. You can find the area in different ways.

a. Tile the section with unit squares. The 1-mile square park is divided up into 10 columns and 10 rows so that one tile is $\frac{1}{10}$ mile by $\frac{1}{10}$ mile.



What is the area of one tile? ..... square mile

How many tiles are in the $\frac{3}{10}$ mile-by-$\frac{3}{10}$ mile section? .....

What is the area of the $\frac{3}{10}$ mile-by-$\frac{3}{10}$ mile section? ..... square mile

b. You can find the area by multiplying the side lengths of the section, $\frac{3}{10} \times \frac{3}{10}$.

Multiply the fractions to find the area.

$\frac{3}{10} \times \frac{3}{10} =$ ..... The area of the section is ..... square mile.

3 REFLECT

Look at the area model and equation for $\frac{3}{10} \times \frac{3}{10}$. Explain how the numerators and denominators in the equation are related to the squares in the area model.

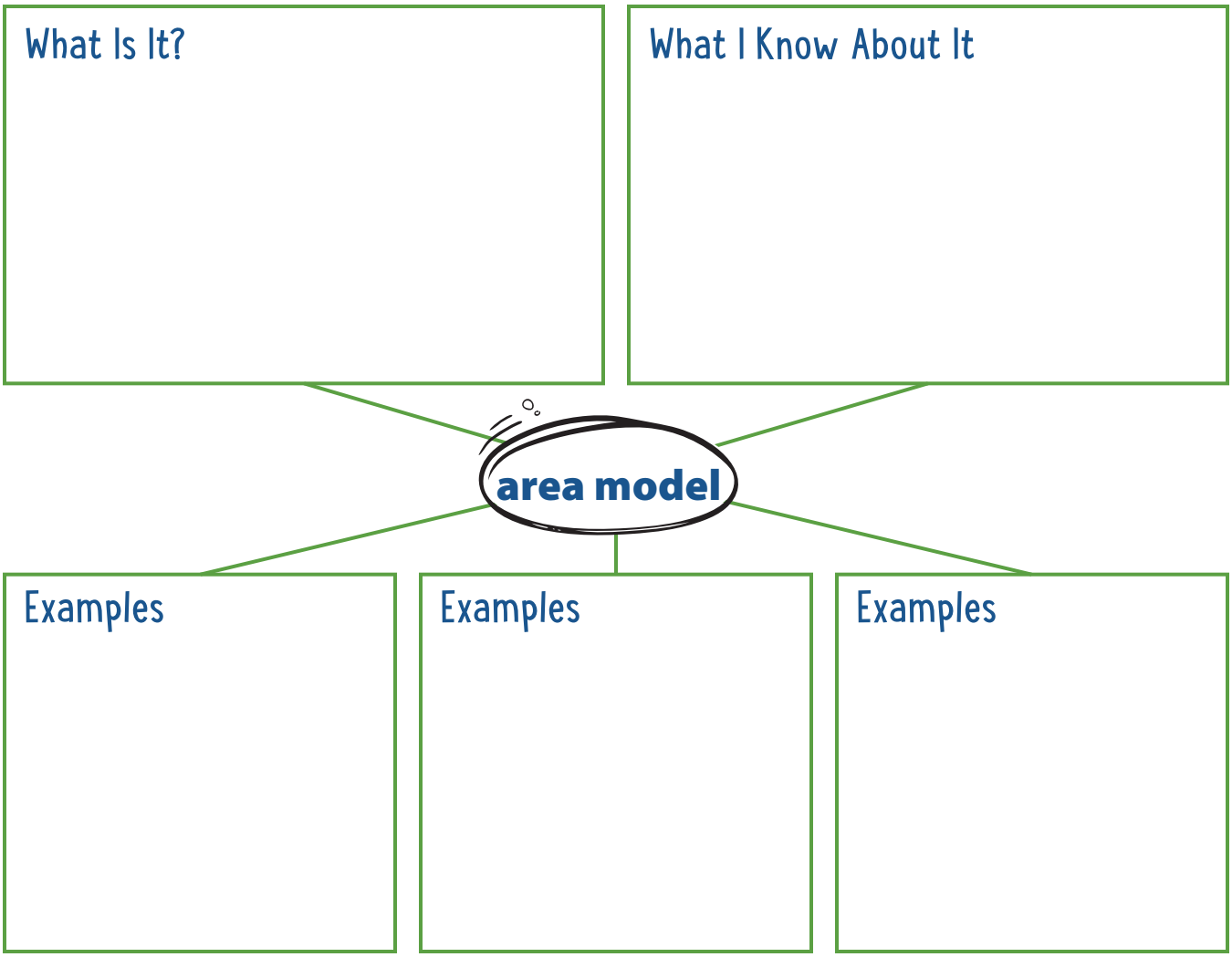
.....

.....

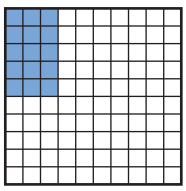
.....

Prepare for Multiplying Fractions to Find Area

1 Think about what you know about the area of a rectangle. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 The area of the whole square at the right is 1 square mile. What is the area of the shaded part of the square?



Solution .....

3 Solve the problem. Show your work.

Mrs. Beaker designs a 1-mile square park. She makes a square with $\frac{6}{10}$-mile sides in her park for sports fields. How many square miles of the park does she use for sports fields?

Solution

4 Check your answer. Show your work.

Develop Multiplying Unit Fractions to Find Area

Read and try to solve the problem below.

Titus is making flashcards to use for vocabulary review. He cuts out rectangular cards that are $\frac{1}{2}$ foot long and $\frac{1}{4}$ foot wide. What is the area of one card in square feet?

TRY IT

Math Toolkit

- fraction models
- multiplication models
- grid paper
- index cards
- cardstock
- rulers
- scissors

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

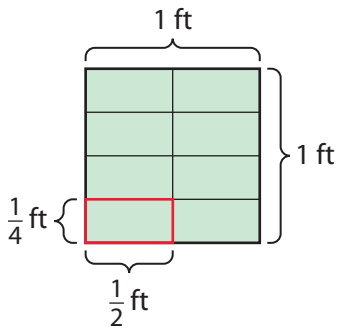
Explore different ways to understand multiplying two unit fractions to find area.

Titus is making flashcards to use for vocabulary review. He cuts out rectangular cards that are $\frac{1}{2}$ foot long and $\frac{1}{4}$ foot wide. What is the area of one card in square feet?

PICTURE IT

You can understand the problem by picturing how the cards could fill a square.

You can arrange 8 cards, each $\frac{1}{2}$ foot by $\frac{1}{4}$ foot, to form a square. When rectangles fill a square without gaps or overlaps, they are said to *tile* the square.



MODEL IT

You can model the problem with an equation.

Each rectangular flashcard has a length of $\frac{1}{2}$ foot and a width of $\frac{1}{4}$ foot.

$$\text{area} = \text{length} \times \text{width}$$

$$\text{area} = \tfrac{1}{2} \text{ foot} \times \tfrac{1}{4} \text{ foot}$$

$$\tfrac{1}{2} \text{ foot} \times \tfrac{1}{4} \text{ foot} = \left(\tfrac{1}{2} \times \tfrac{1}{4}\right) \text{ square foot}$$

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply two unit fractions to find area.

1. Why can a sheet of paper that is 1 square foot be tiled by 4 rows of 2 flashcards?

2. What portion of the 1-square-foot sheet of paper is represented by each flashcard?

   _____ of the sheet of paper, or _____ of 1 square foot

3. Look at the equations in **Model It**. What product of unit fractions can you use to find the area of one flashcard? _____

4. Multiply the denominators of the unit fractions. How does the product relate to the size of the units compared to the size of the whole model?

5. Multiply the numerators of the unit fractions. How does the product relate to the number of outlined parts of the model?

6. Explain how to find the area of one flashcard.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding the area of a rectangle with fractional side lengths? Explain.

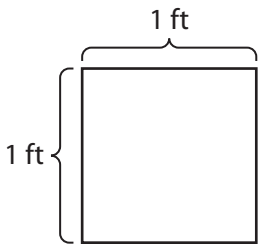
APPLY IT

Use what you just learned to solve these problems.

8 What is the area of a rectangular paper strip with a length of $\frac{1}{3}$ yard and width of $\frac{1}{6}$ yard? Use an area model and an equation to show your work.

Solution

9 Marnie has a square picture with an area of 1 square foot. Draw lines in the model below to show one way to tile a 1-foot square with rectangular tiles that each have an area of $\frac{1}{12}$ square foot. What are the length and width of your tile?



Solution

10 What is the area of a rectangle that has a length of $\frac{1}{4}$ inch and a width of $\frac{1}{8}$ inch?

Ⓐ $\frac{1}{12}$ square inch

Ⓑ $\frac{2}{12}$ square inch

Ⓒ $\frac{1}{32}$ square inch

Ⓓ $\frac{2}{32}$ square inch

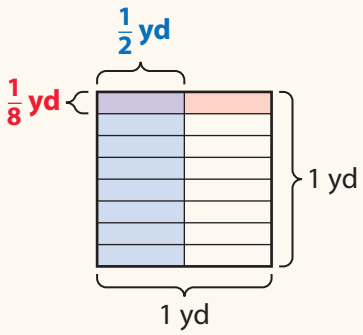
Practice Multiplying Unit Fractions to Find Area

Study the Example that shows multiplying unit fractions to find area. Then solve problems 1–5.

EXAMPLE

A sheet of cardboard that measures 1 yard on each side is cut into rectangular cards that are $\frac{1}{8}$ yard wide and $\frac{1}{2}$ yard long. What is the area of each card?

You can model the problem with a picture as shown.



You also can model the problem with an equation.

$$\text{area} = \frac{1}{2} \times \frac{1}{8}$$
$$= \frac{1 \times 1}{2 \times 8}$$
$$= \frac{1}{16}$$

The area of each card is $\frac{1}{16}$ square yard.

1 Suppose the length of each card in the Example is shortened to $\frac{1}{4}$ yard. Will the area of each card now be greater than or less than $\frac{1}{16}$ square yard? Explain.

2 Which expression represents the area of a rectangular card described in problem 1?

Ⓐ $\frac{1}{2} \times \frac{1}{4}$

Ⓑ $\frac{1}{2} \times \frac{1}{8}$

Ⓒ $\frac{1}{4} \times \frac{1}{8}$

Ⓓ $\frac{1}{4} \times \frac{1}{16}$

3 What is the area of a rectangular card that is $\frac{1}{8}$ yard wide and $\frac{1}{4}$ yard long? Show your work.

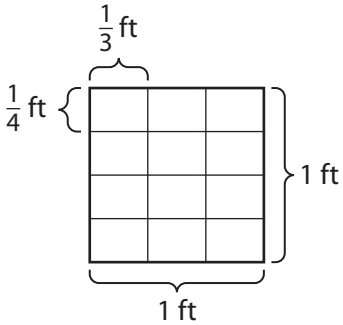
Solution

4 Mr. Von's 5th-grade class is going on a field trip. Each student is given a rectangular name card to wear that is $\frac{1}{4}$ foot wide and $\frac{1}{3}$ foot long.

Shade the model to find the area of each name card.

Complete the equation.

$\frac{1}{4}$ foot $\times \frac{1}{3}$ foot $= \frac{\square}{\square}$ square foot



5 Signs for project displays are cut from pieces of poster board that measure 1 yard on each side. Each rectangular sign is $\frac{1}{3}$ yard long and $\frac{1}{9}$ yard wide. How many signs can be cut from one piece of poster board? What is the area of each sign? Show your work.

Solution

Develop Tiling a Rectangle to Find Area

Read and try to solve the problem below.

A rectangular postage stamp has a length of $\frac{3}{2}$ inches and a width of $\frac{3}{4}$ inch. What is the area of the stamp in square inches?

TRY IT

Math Toolkit

- fraction models
- multiplication models
- half-inch grid paper
- index cards
- rulers

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I started by . . .

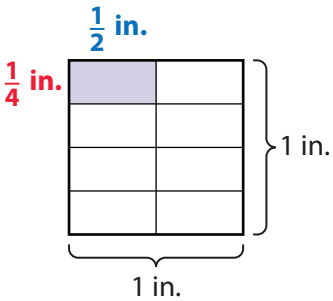
Explore different ways to understand modeling the area of a rectangle through tiling and equations.

A rectangular postage stamp has a length of $\frac{3}{2}$ inches and a width of $\frac{3}{4}$ inch. What is the area of the stamp in square inches?

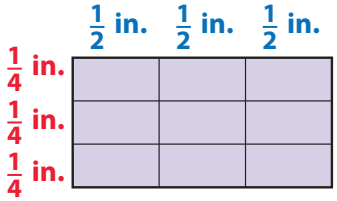
PICTURE IT

You can picture tiling the rectangular stamp with smaller rectangles that have unit fractions as side lengths.

You can tile a unit square with rectangles that have unit fractions as side lengths. Each rectangular tile is $\frac{1}{2}$ inch by $\frac{1}{4}$ inch.



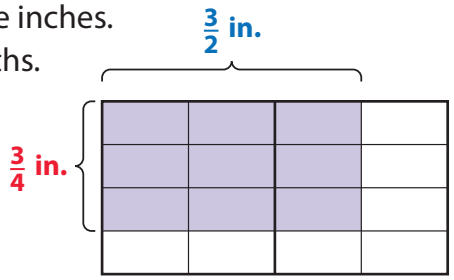
You can also use $\frac{1}{2}$ inch-by-$\frac{1}{4}$ inch tiles to tile a rectangular stamp with length $\frac{3}{2}$ inches and width $\frac{3}{4}$ inch.



MODEL IT

You can model the area of the rectangular stamp with an equation.

Think of the stamp as part of two whole square inches. Use the area formula to multiply the side lengths.



area = length × width

area = $\frac{3}{2}$ inches × $\frac{3}{4}$ inch

$\frac{3}{2}$ inches × $\frac{3}{4}$ inch = $\left(\frac{3}{2} \times \frac{3}{4}\right)$ square inches

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use tiling or equations to find area.

1. Use the unit square in **Picture It** to explain how to find the area of one tile.

2. Look at the model of the stamp in **Picture It**. Explain why nine $\frac{1}{2}$ inch-by-$\frac{1}{4}$ inch rectangles tile the $\frac{3}{2}$ inches-by-$\frac{3}{4}$ inch stamp.

3. Write an equation that uses the area of one tile to find the area of the stamp.

   _____ square inches = _____ square inches

4. Now look at the area formula equation in **Model It**. Complete the equation to find the area of the stamp as shown in this model.

   $\frac{3}{2} \times \frac{3}{4} = \frac{\square \times \square}{\square \times \square} =$ _____ The area is _____ square inches.

5. Does using the area formula equation result in the same area as you found by tiling the rectangle? Why?

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding the area of a rectangle with fractional side lengths? Explain.

_____

_____

_____

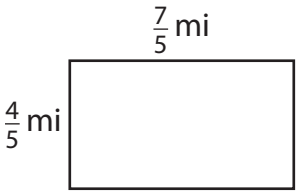
APPLY IT

Use what you just learned to solve these problems.

7 Bernice's rectangular math workbook is $\frac{2}{3}$ foot wide and $\frac{5}{6}$ foot long. What is the area of a page in her workbook? Show your work.

Solution

8 Show one way to use tiles to find the area of the rectangle below. What are the length and width of one of your tiles? What is the area of the rectangle?



Solution

9 John's rectangular poster is $\frac{7}{4}$ yards in length and $\frac{2}{3}$ yard in width. What is the area of John's poster?

Ⓐ $\frac{2}{3}$ square yard

Ⓑ $\frac{14}{12}$ square yards

Ⓒ $\frac{9}{7}$ square yards

Ⓓ $\frac{7}{4}$ square yards

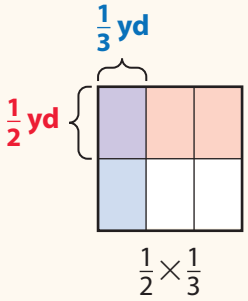
Practice Tiling a Rectangle to Find Area

Study the Example that shows tiling a rectangle to find its area. Then solve problems 1–6.

EXAMPLE

What is the area of a rectangle that is $\frac{1}{2}$ yard wide and $\frac{4}{3}$ yards long?

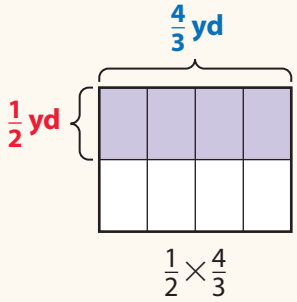
The top area model shows that $\frac{1}{2}$ yard $\times$ $\frac{1}{3}$ yard $=$ $\frac{1}{6}$ square yard.



The bottom model uses the same $\frac{1}{6}$-square-yard parts to show an area that is $\frac{1}{2}$ yard $\times$ $\frac{4}{3}$ yards.

Four $\frac{1}{6}$-square-yard parts are shaded purple.

$\frac{1}{2}$ yard $\times$ $\frac{4}{3}$ yards $=$ $\frac{4}{6}$ square yard

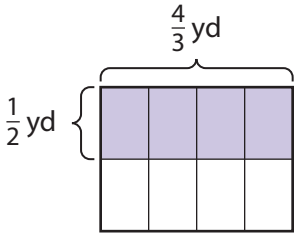


1. How many $\frac{1}{2}$-yard lengths are in 1 yard? .....

2. How many $\frac{1}{3}$-yard lengths are in 1 yard? .....

3. Draw a line around the part of the model from the Example that represents 1 square yard.

   Does $\frac{4}{6}$ square yard cover more area or less area than 1 square yard? Explain.



4 Danah has a rectangular strawberry patch in her garden. Its border is $\frac{7}{8}$ yard wide and $\frac{3}{2}$ yards long. Use a visual model to find the area of Danah's strawberry patch. Then write an equation to describe your model. Show your work.

Solution

5 Danah is planting a second rectangular strawberry patch and wants it to have an area of exactly 1 square yard. Which of the following could be the width and length of its borders? Select all that apply.

Ⓐ $\frac{1}{2}$ yard wide and $\frac{3}{2}$ yards long

Ⓑ $\frac{2}{3}$ yard wide and $\frac{3}{2}$ yards long

Ⓒ $\frac{4}{5}$ yard wide and $\frac{5}{4}$ yards long

Ⓓ $\frac{2}{3}$ yard wide and $\frac{6}{4}$ yards long

Ⓔ $\frac{3}{4}$ yard wide and $\frac{12}{8}$ yards long

6 Look at problem 5. If Danah wants the area of her rectangular strawberry patch to be exactly 1 square yard, can the length of the strawberry patch be greater than 1 yard? Explain.

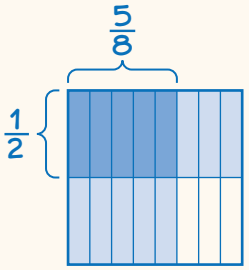
Refine Multiplying Fractions to Find Area

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Rachel is designing a newspaper ad. The ad will include a rectangular piece of art whose dimensions are $\frac{5}{8}$ inch long and $\frac{1}{2}$ inch wide. How many square inches of space will the art cover?

Look at how you could show your work using a unit square area model and an equation.



$$\frac{5}{8} \times \frac{1}{2} = \frac{5 \times 1}{8 \times 2}$$

Solution

$\frac{1}{8}$ inch $\times$ $\frac{1}{2}$ inch is $\frac{1}{16}$ square inch. How many one-sixteenth square inches are shown in the model?

PAIR/SHARE

How can you write $\frac{5}{8} \times \frac{1}{2}$ as a product of unit fractions and whole numbers?

APPLY IT

1 What is the area of a rectangle with a length of $\frac{1}{2}$ yard and a width of $\frac{11}{6}$ yards? Write an equation to represent your solution. Show your work.

Solution

How can you represent a fractional side length with an area model?

PAIR/SHARE

Find the area of a rectangle with side lengths of $\frac{3}{4}$ yard and $\frac{6}{5}$ yards. How is the model different?

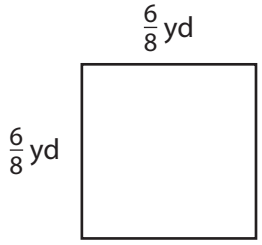
2 Kwame is designing a poster that has an area of 1 square foot. He is going to paste a photo collage on a rectangular section of the poster that is $\frac{1}{3}$ foot wide and $\frac{3}{4}$ foot long. What part of a square foot will the photo collage cover? Show your work.

If I draw a square to represent a square foot, how can I represent thirds and fifths on the square?

PAIR/SHARE
Write an equation to represent your model. Explain the meaning of the numerators.

Solution

3 What is the area of the square?



Ⓐ $\frac{36}{64}$ square yard

Ⓑ $\frac{12}{16}$ square yard

Ⓒ $\frac{64}{36}$ square yards

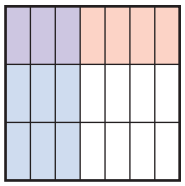
Ⓓ $\frac{12}{8}$ square yards

Think about the size of the two fractions. Will the product of the fractions be greater than 1 or less than 1?

Ollie chose Ⓓ as the correct answer. How did he get that answer?

PAIR/SHARE
Does Ollie's answer make sense?

4 The square at the right represents 1 square unit.



Which expression represents the area of the purple section?

Ⓐ $\frac{7}{3} \times \frac{3}{1}$ square units

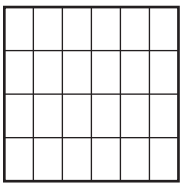
Ⓑ $\frac{3}{7} \times \frac{1}{3}$ square units

Ⓒ $\frac{1}{7} \times \frac{1}{3}$ square units

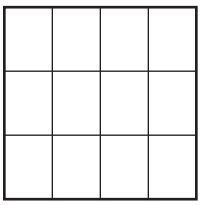
Ⓓ $\frac{7}{3} \times \frac{1}{3}$ square units

5 Fill in the missing numbers to make the equation true. Then complete the area model to check your answer.

$$\frac{1}{6} \times \frac{\square}{\square} = \frac{1}{24}$$



6 Which products could you find by shading the model below?



Ⓐ $\frac{3}{4} \times \frac{1}{3}$

Ⓑ $\frac{1}{3} \times \frac{1}{6}$

Ⓒ $\frac{2}{3} \times \frac{1}{4}$

Ⓓ $\frac{5}{3} \times \frac{1}{4}$

Ⓔ $\frac{3}{4} \times \frac{3}{4}$

7 Draw an area model to represent the expression $\frac{5}{4}$ miles $\times \frac{4}{5}$ mile.

What are the dimensions of one of the rectangular tiles in your model?

8 Explain how to find the area of the model you drew in problem 7. Then find the area.

9 MATH JOURNAL

Find the area of a rectangle $\frac{5}{3}$ units in length and $\frac{3}{4}$ units in width. Show and explain how to find the area.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.